BST 760: Advanced Regression
Breheny

Assignment 2
Due: Thursday, January 27

1. Express the equalities from assignment 1, parts (c)-(e), in matrix form (*i.e.*, without any summation symbols).
2. Suppose $\mathbf{D}$ is a diagonal matrix with diagonal elements $\{d_i\}$. Find $\mathbf{D}^{-1}$.
3. Refer to slide 12 of the January 20th notes.
   (a) Under what condition(s) would the columns of $\mathbf{X}$ be orthogonal to each other?
   (b) Suppose that $\mathbf{X}$ is orthogonal, in the sense that $\mathbf{X}^T\mathbf{X} = \mathbf{I}$. How does this simplify the estimates of the regression coefficients α and β?
4. Suppose $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to each other. Are $-\mathbf{u}$ and $2\mathbf{v}$ orthogonal to each other?
5. Show that if $\mathbf{A}$ is invertible, then its inverse is unique. (Hint: Show that if $\mathbf{B}$ and $\mathbf{B}^*$ are both inverses of $\mathbf{A}$, then $\mathbf{B} = \mathbf{B}^*$)
6. Solve for $\mathbf{X}$ in the following equations (you may assume that all matrices are conformable and that all the inverses you need exist):
   (a) $\mathbf{AX} + \mathbf{B} = \mathbf{BX}$
   (b) $\mathbf{XAB} - \mathbf{BA} = 0$
   (c) $\mathbf{AXB} = \mathbf{C}$
7. Let
$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$
   (a) Find $\mathbf{A}^T\mathbf{A}$. You may use a computer, although it wouldn't hurt to try it by hand and confirm that you get the same answer.
   (b) Find $(\mathbf{A}^T\mathbf{A})^{-1}$. If you really want to, you can try solving this by hand, but I would recommend using a computer.
   (c) Is $\mathbf{A}^T\mathbf{A}$ positive definite?
   (d) What is the rank of $\mathbf{A}$?