Survival Data Analysis (BIOS:7210)
Breheny

Assignment 5

Due: Thursday, October 17

1. In class, we derived score, Wald, and likelihood ratio test approaches for the exponential distribution and applied them to the Pike et al. (1966) rat data. As we did in class, use years as the unit of time (the data is provided in days). For full credit, derive a closed form solution for the score intervals.
   (a) Provide confidence intervals for λ based on the score, Wald, and likelihood ratio approaches.
   (b) Consider the transformation $\gamma = \log \lambda$. What is the score function with respect to γ?
   (c) What is the information with respect to γ? Comment on the observed information evaluated at $\hat{\gamma}$; what property does it have?
   (d) Provide score, Wald, and likelihood ratio confidence intervals for γ.
   (e) Transform the intervals from (d) to yield confidence intervals for λ; how do they compare with the intervals in (a)?
   (f) Test the hypothesis that $\gamma = 0$ (i.e., that $\lambda = 1$) using the score and Wald approaches. How do the results compare with the p-values we obtained in class?
   (g) Consider the transformation $\alpha = \lambda^{1/3}$. Derive the score and observed information with respect to α.
   (h) Take the third derivative of the log-likelihood with respect to α and evaluate it at the MLE $\hat{\alpha}$. What happens?
   (i) Plot the log-likelihood as a function of α, along with the Wald approximation. Comment on the accuracy of the Wald approach's Taylor series approximation compared to the plot on slide 22 of the "Likelihood-based inference: Single Parameter" notes.
   (j) Provide score, Wald, and likelihood ratio confidence intervals for α.
   (k) Transform the intervals from (j) to yield confidence intervals for λ; how do they compare with each other?
   (l) Test the hypothesis that $\alpha = 1$ (i.e., that $\lambda = 1$) using the score and Wald approaches. How do the results compare with the p-values we obtained in class and those from (f)?
2. We stated in class that $\boldsymbol{\mathcal{I}} = -\mathbb{E}(\nabla \mathbf{u})$. Prove that this holds for independent and identically distributed right-censored data, where the censoring time c is fixed. You will want to prove this in two parts: one for the diagonal elements and one for the off-diagonal elements. Hint: note that the contribution to the likelihood depends on whether $t < c$ or not; thus, you will want to break up your expectation into these cases.
3. Show that $[\mathbf{I}(\hat{\boldsymbol{\theta}})^{-1}]_{jj} \geq [\mathbf{I}(\hat{\boldsymbol{\theta}})_{jj}]^{-1}$. In what scenario is it possible for these two quantities to be equal?