Survival Data Analysis (BIOS:7210)
Breheny

Assignment 5

Due: Thursday, October 11

1. In class, we derived score, Wald, and likelihood ratio test approaches for the exponential distribution and applied them to the Pike et al. (1966) rat data. As we did in class, use years as the unit of time (the data is provided in days). For full credit, derive a closed form solution for the score intervals.
   (a) Provide confidence intervals for λ based on the score, Wald, and likelihood ratio approaches.
   (b) Consider the transformation $\gamma = \log \lambda$. What is the score function with respect to γ?
   (c) What is the information with respect to γ? Comment on the observed information evaluated at $\hat{\gamma}$; what property does it have?
   (d) Provide score, Wald, and likelihood ratio confidence intervals for γ.
   (e) Transform the intervals from (d) to yield confidence intervals for λ; how do they compare with the intervals in (a)?
   (f) Test the hypothesis that $\gamma = 0$ (i.e., that $\lambda = 1$) using the score and Wald approaches. How do the results compare with the p-values we obtained in class?
   (g) Consider the transformation $\alpha = \lambda^{1/3}$. Derive the score and observed information with respect to α.
   (h) Take the third derivative of the log-likelihood with respect to α and evaluate it at the MLE $\hat{\alpha}$. What happens?
   (i) Plot the log-likelihood as a function of α, along with the Wald approximation. Comment on the accuracy of the Wald approach's Taylor series approximation compared to the plot on slide 25 of the September 20 notes.
   (j) Provide score, Wald, and likelihood ratio confidence intervals for α.
   (k) Transform the intervals from (j) to yield confidence intervals for λ; how do they compare with each other?
   (l) Test the hypothesis that $\alpha = 1$ (i.e., that $\lambda = 1$) using the score and Wald approaches. How do the results compare with the p-values we obtained in class and those from (f)?
2. Prove, as we stated in class, that $\boldsymbol{\mathcal{I}} = -\mathbb{E}(\nabla \mathbf{u})$. The univariate derivation we went over in class suffices for the diagonal elements, but needs to be modified slightly for the off-diagonal elements.
3. Show that $[\mathbf{I}(\hat{\boldsymbol{\theta}})^{-1}]_{jj} \geq [\mathbf{I}(\hat{\boldsymbol{\theta}})_{jj}]^{-1}$. In what scenario is it possible for these two quantities to be equal?