Theorem: Suppose $f_n \to f$ uniformly, with f_n continuous for all n. Then $f_n(\mathbf{x}) \to f(\mathbf{x}_0)$ as $\mathbf{x} \to \mathbf{x}_0$.

Proof. Let $\epsilon > 0$.

①	$\exists N : n > N \implies \sup_x \lvert f_n(\mathbf{x}) - f(\mathbf{x}) \rvert < \frac{\epsilon}{2}$	Def. Uniform convergence
②	$\exists \delta : \lVert \mathbf{x} - \mathbf{x}_0 \rVert < \delta \implies \lvert f(\mathbf{x}) - f(\mathbf{x}_0) \rvert < \frac{\epsilon}{2}$	f is continuous

Therefore, for $n > N$ and any $\mathbf{x} \in N_\delta(\mathbf{x}_0)$, we have

$$\begin{aligned}
\lvert f_n(\mathbf{x}) - f(\mathbf{x}_0) \rvert &= \lvert f_n(\mathbf{x}) - f(\mathbf{x}) + f(\mathbf{x}) - f(\mathbf{x}_0) \rvert \\
&\leq \lvert f_n(\mathbf{x}) - f(\mathbf{x}) \rvert + \lvert f(\mathbf{x}) - f(\mathbf{x}_0) \rvert && \text{Triangle inequality} \\
&< \sup_x \lvert f_n(\mathbf{x}) - f(\mathbf{x}) \rvert + \tfrac{\epsilon}{2} && \text{②} \\
&< \epsilon && \text{①}
\end{aligned}$$

□