

Likelihood Theory and Extensions (BIOS:7110)
Breheny

Assignment 3

Due: Monday, September 13

1. *O*-notation proofs. Prove the following results:

- (a) $O(1)o(1) = o(1)$.
- (b) $\{1 + o(1)\}^{-1} = O(1)$.
- (c) $o\{O(1)\} = o(1)$.

2. *Exponential Taylor series*.

- (a) Show that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

- (b) What is the infinite series for e^{ax} ?
- (c) What is the infinite series for b^x ?
- (d) Let $f : \mathbb{R}^d \rightarrow \mathbb{R}$. What is the second-order Taylor series for $f(\mathbf{x}) = \exp(\mathbf{a}^\top \mathbf{x})$ about $\mathbf{x} = \mathbf{0}$? Give both the *o*-notation and Lagrange forms.
- (e) Suppose $\mathbf{a} = [2 \quad -1]^\top$ and $\mathbf{x} = [1 \quad 1]^\top$. Find the point \mathbf{x}^* on the line segment connecting \mathbf{x} and $\mathbf{0}$ that satisfies the Lagrange form of Taylor's theorem.