Likelihood Theory and Extensions (BIOS:7110)
Breheny

Assignment 7

Due: Monday, October 26

1. *Neyman-Scott problem.* Suppose Y_{i1} and Y_{i2} are an iid sample from $\mathrm{N}(\mu_i, \sigma^2)$, for $i = 1, \ldots, n$.
   (a) Find the maximum likelihood estimate of σ^2.
   (b) Show that the MLE of σ^2 is not consistent.
   (c) Why does our theorem about the consistency of maximum likelihood estimates not hold in this situation?
   (d) Find a consistent estimator of σ^2.

2. Suppose $X_1, \ldots, X_n$ are an iid sample with density

$$p(x|\theta) = \theta x^{\theta-1} 1\{x \in (0,1)\}, \quad \Theta = (0, \infty).$$

   Find the MLE and its limiting distribution.

3. *Combination of Poisson variables.* Suppose in one month, a public health department records X cases of a new disease. At the end of the month, it is discovered that this disease is really two different diseases with similar symptoms. In month two, data recording practices are changed and the number of cases of each disease, Y_1 and Y_2, is recorded separately. For the purposes of the problem, assume that the number of cases of disease i in a month follows a Poisson distribution with rate λ_i, that cases of the two diseases arise independently, and that the number of cases in one month is independent of the number of cases in the next month.
   (a) Find the MLE of λ_1 and λ_2.
   (b) Suppose λ_1 and λ_2 are large enough that we feel comfortable applying the central limit theorem to (X, Y_1, Y_2). What is the approximate variance of $\hat{\lambda}_1$ as a function of λ_1 and λ_2?