Lab 6

February 25-26, 2020

In class last week, we discussed and practiced the rules of probability. In today's lab, we will review these concepts.

Properties of Probability

$$\text{Addition Rule: } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{Complement Rule: } P(A^C) = 1 - P(A)$$

$$\text{Multiplication Rule: } P(A \cap B) = P(A)P(B|A)$$

$$\text{Law of Total Probability: } P(A) = P(A \cap B) + P(A \cap B^c)$$

$$\text{Bayes' Rule: } P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A)+P(A^c)P(B|A^c)}$$

Independence VS Mutually Exclusive

These two terms are very easily confused, but need to be kept separate as they mean two different things. Two outcomes are said to be independent if the probability of the first outcome does not change the probability of the second outcome.

$$\text{Independent: } P(A|B) = P(A)$$

This independence works both ways as the outcome of A will also not effect the outcome of B. Mutually exclusive on the other hand, is the idea that the two outcomes cannot occur simultaneously. That is, the interesection of the two events is always equal to zero. In terms of a venn diagram the two cirlces will never overlap.

$$\text{Mutually Exclusive: } P(A \cap B) = 0$$

Practice Probability Examples

1. On the Iowa Football Team there are exactly 99 players on the team. Of them 32 are freshman, 12 are sophmores, 34 are juniors, and 21 are seniors.

A. Are the years of study mutually exclusive?

B. If a football player is selected at random what is the probability that he is not a senior?

C. If you select a random football player and it is given that he is not a senior what is the probability of him being a sophmore?

D. What is the probability that I pull two juniors out randomly with replacement? Without replacement?

What assumption changed based on the question asked?

2. Consider the following table regarding the periodontal status of individuals and their gender. Periodontal status refers to gum disease where individuals are classified as either healthy, have gingivitis, or have periodontal disease.

	Periodontal Status			
Gender	Healthy	Gingivitis	Perio	Total
Male				
Female				
Total				

Fill in the following table with the given information:

The probability that a person is male is 0.3749,

The probability that a person is both male and healthy is 0.1429,

The probability that a person is both male and has Perio is 0.1167,

The probability of being healthy is 0.4672,

The probability that a person is female and has Perio 0.1147

A. Calculate the probability that the particpant is male or healthy.

B. What is the probability of being female and having either gingivitis or periodontal disease?

C. What is the probability of an individual having periodontal disease given that she is female?

3. In a certain region, one in every thousand people (0.001) is infected by the HIV virus that causes AIDS. Tests for presence of the virus are fairly accurate but not perfect.If someone actually has HIV, the probability of testing positive is 0.95.

A. Let H denote the event of having HIV, and T the event of testing positive. Express the information that is given in the problem in terms of the events H and T.

B. Find the probability that someone chosen at random from the population has HIV and tests positive. *Interpret this as a rate per 10,000 people*

C. If someone has HIV, what is the probability of testing negative?

Screening Test Examples

4. Scenario (from Gigerenzer 2007): The prevalence of breast cancer in women is 1%. A mammography has sensitivity of 90% (90% of women with breast cancer will test positive) and a false positive rate of 9%.

What is the probability a person has the disease given that they tested positive (use Bayes' rule).

A. To set up this equation, first write the information provided in the form of probability notation and identify the corresponding proportions. You should do this for prevalence of cancer, prevalence of no cancer, sensitivity, specificity, and false positive and false negative rates.

B. Now modify the Bayes' rule formula to match the notation you wrote in part (a) to find the probability of breast cancer given that the test is positive.

C. Finally plug in and calculate the answer. Interpret the results.