

Lab 6

February 20-21, 2018

In class last week we discussed and practiced the rules of probability. In today's lab we will review these concepts and give you some different ways to look at probability calculations such as tree diagrams.

Properties of Probability

$$\text{Addition Rule: } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{Complement Rule: } P(A^C) = 1 - P(A)$$

$$\text{Multiplication Rule: } P(A \cap B) = P(A)P(B|A)$$

$$\text{Law of Total Probability: } P(A) = P(A \cap B) + P(A \cap B^c)$$

$$\text{Bayes' Rule: } P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A^c)P(B|A^c)}$$

Independence VS Mutually Exclusive

These two terms are very easily confused, but need to be kept separate as they mean two different things. Two outcomes are said to be independent if the probability of the first outcome does not change the probability of the second outcome.

$$\text{Independent: } P(A|B) = P(A)$$

This independence works both ways as the outcome of A will not effect the outcome of B also. Mutually exclusive on the other hand is the idea that the two outcomes cannot occur simultaneously. That is the intersection of the two events is always equal to zero. In terms of a venn diagram the two circles will never overlap.

$$\text{Mutually Exclusive: } P(A \cap B) = 0$$

Tree Diagrams to Help With Problems

When doing probability examples sometimes it may come in handy to use tree diagrams. you start with a probability of one at the top and fraction off probabilities as you go down. In some of the problems below we will work out how to do them using a tree diagram.

Practice Probability Examples

1. On the Iowa Football Team there are exactly 99 players on the team. Of them 32 are freshman, 12 are sophomores, 34 are juniors, and 21 are seniors.

Are the variables of interest mutually exclusive?

- Set up a tree diagram.
- If a football player is selected at random what is the probability that he is not a senior?
- If you select a random football player and it is given that he is not a senior what is the probability of him being a sophomore?
- What is the probability that I pull two juniors out randomly with replacement? without replacement?
What assumption changed based on the question asked?

2. Consider the following table regarding the periodontal status of individuals and their gender. Periodontal status refers to gum disease where individuals are classified as either healthy, have gingivitis, or have periodontal disease.

		periodontal status			Total
		healthy	gingivitis	perio	
GENDER	male	1143	929	937	3009
	female	2607	1490	921	5018
Total		3750	2419	1858	8027

Figure 1: Dental Health

- Calculate the probability that the participant is male or healthy.
- What is the probability of being female and having either gingivitis or periodontal disease?

3. In a certain region, one in every thousand people (0.001) is infected by the HIV virus that causes AIDS. Tests for presence of the virus are fairly accurate but not perfect. If someone actually has HIV, the probability of testing positive is 0.95. If someone actually has HIV, the probability of testing positive is 0.95.

- Let H denote the event of having HIV, and T the event of testing positive. Express the information that is given in the problem in terms of the events H and T.
- Find the probability that someone chosen at random from the population has HIV and tests positive.
Interpret this as a rate per 10,000 people
- If someone has HIV, what is the probability of testing negative?

Screening Test Examples

4. Scenario (from Gigerenzer 2007): The prevalence of breast cancer in women is 1%. A mammography has sensitivity of 90% (90% of women with breast cancer will test positive) and a false positive rate of 9%.

What is the probability a person has the disease given that they tested positive (use Bayes' rule).

A. To set up this equation, first write the information provided in the form of probability notation and identify the corresponding proportions. You should do this for prevalence of cancer, prevalence of no cancer, sensitivity, specificity, and false positive and false negative rates.

B. Now modify the Bayes' rule formula to match the notation you wrote in part (a) to find the probability of disease given that the test is positive.

C. Finally plug in and calculate the answer. How would you do this with the tree diagram? Interpret the results.

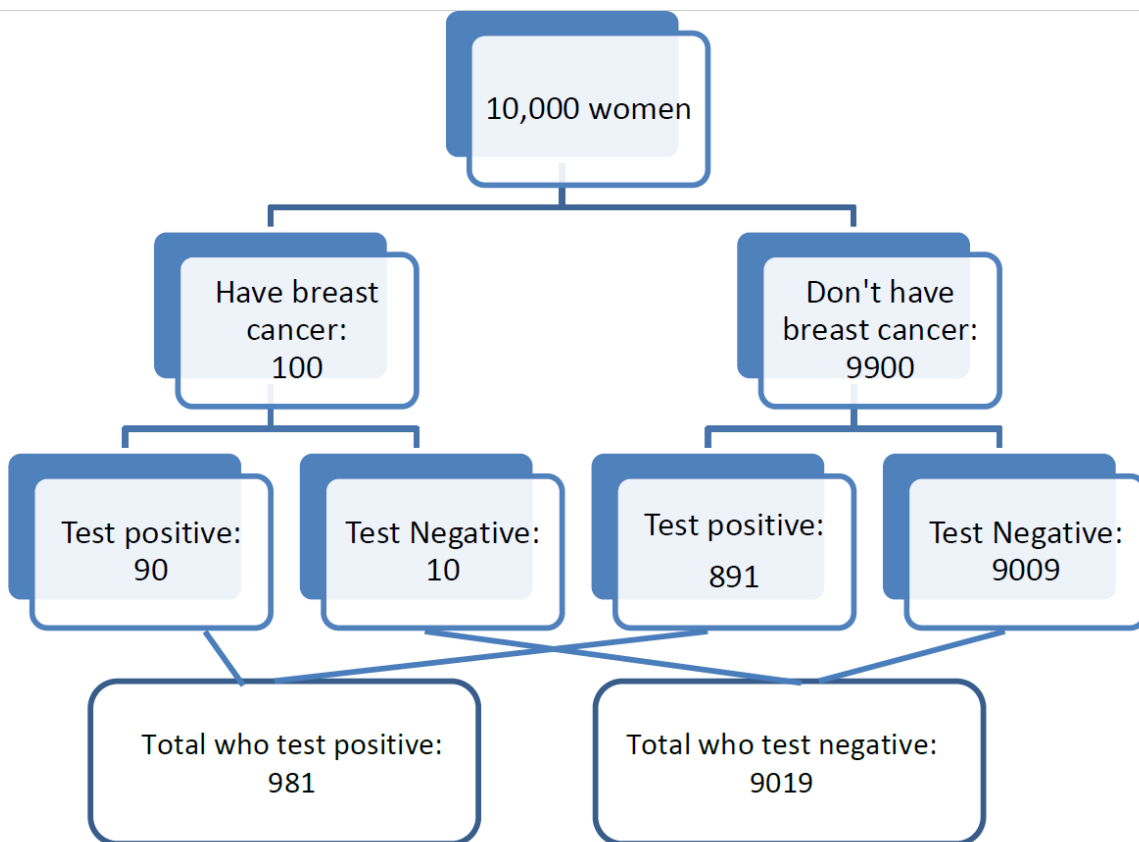


Figure 2:

Question 2 & 3 sourced from Module 6 and 7:

<http://bolt.mph.ufl.edu/6050-6052/unit-3/>