

# Correlation

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# Introduction

- Box plots are a way to examine the relationship between a continuous variable and a categorical variable
- In lab, we saw bar charts as a way of comparing two (or more) categorical variables
- Now, we will discuss how to summarize and illustrate the relationship between two continuous variables

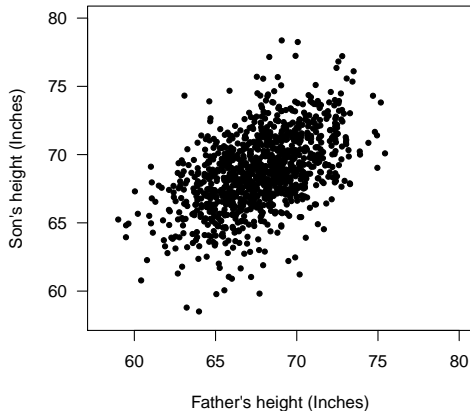
## Pearson's height data

- Statisticians in Victorian England were fascinated by the idea of quantifying hereditary influences
- Two of the pioneers of modern statistics, the Victorian Englishmen Francis Galton and Karl Pearson were quite passionate about this topic
- In pursuit of this goal, they measured the heights of 1,078 fathers and their (fully grown) sons

# The scatter plot

- As we've mentioned, it is important to plot continuous data – this is especially true when you have two continuous variables and you're interested in the relationship between them
- The most common way to plot the relationship between two continuous variables is the *two-way scatter plot*
- Scatter plots are created by setting up two continuous axes, then creating a dot for every pair of observations

# Scatter plot of Pearson's height data



# Observations about the scatter plot

- Taller fathers tend to have taller sons
- The scatter plot shows how strong this association is – there is a tendency, but there are plenty of exceptions

## Standardizing a variable

- Before we summarize this relationship numerically, we must discuss the idea of *standardizing* a variable
- In Pearson's height data, one of the sons measured 63.2 inches tall
- Because the average height of the sons in the sample was 68.7 inches, another way of describing his height is to say that he was 5.5 inches below average
- Furthermore, because the standard deviation of the sons was 2.8 inches, yet another way of describing his height is to say that he was 1.9 standard deviations below the average

## The standardization formula

- Putting this into a formula, we standardize an observation  $x_i$  by subtracting the average and dividing by the standard deviation:

$$z_i = \frac{x_i - \bar{x}}{SD_x}$$

where  $\bar{x}$  and  $SD_x$  are the mean and standard deviation of the variable  $x$

- One virtue of standardizing a variable is interpretability:
  - If someone tells you that the concentration of urea in your blood is 50 mg/dL, that likely means nothing to you
  - On the other hand, if you are told that the concentration of urea in your blood is 4 standard deviations above average, you can immediately recognize this as a very high value



## More benefits of standardization

- If you standardize all of the observations in your sample, the resulting variable will be “standardized” in the sense of having mean 0 and standard deviation 1
- Standardization therefore brings all variables onto a common scale – regardless of whether the heights were originally measured in inches, centimeters, or miles, the standardized heights will be identical
- As we will see momentarily, this allows us to study the relationship between two continuous variables without worrying about the scale of measurement
- The concept behind standardization – taking an observation, then subtracting the expected value and dividing by the variability – is fundamental to statistics and we will see variations on this idea many times in this course

# The correlation coefficient

- The summary statistic for describing the strength of association between two variables is the *correlation coefficient*, denoted by  $r$  (and sometimes called Pearson's correlation coefficient)
- The correlation coefficient is always between 1 (perfect positive correlation) and -1 (perfect negative correlation), and can take on any value in between
- A *positive correlation* means that as one variable increases, the other one tends to increase as well
- A *negative correlation* means that as one variable increases, the other one tends to decrease

## Calculating the correlation coefficient

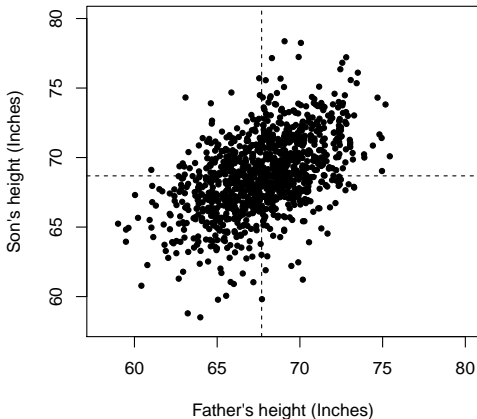
- The correlation coefficient is simply the average of the products of the standardized variables
- In mathematical notation,

$$r = \frac{\sum_{i=1}^n \tilde{x}_i \tilde{y}_i}{n - 1},$$

where  $\tilde{x}_i$  and  $\tilde{y}_i$  are the standardized values of  $x$  and  $y$  (i.e., the  $z_i$ 's computed separately for variable  $x$  and variable  $y$ )

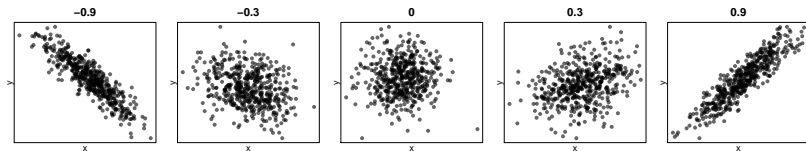
- Note: The “ $n$  versus  $n - 1$ ” issue has nothing to do with correlation; however, if  $n - 1$  is used when standardizing, it must be used again here

## Meaning behind the correlation coefficient formula



For this data,  
 $r = 0.50$

# The correlation coefficient and the scatter plot



## More about the correlation coefficient

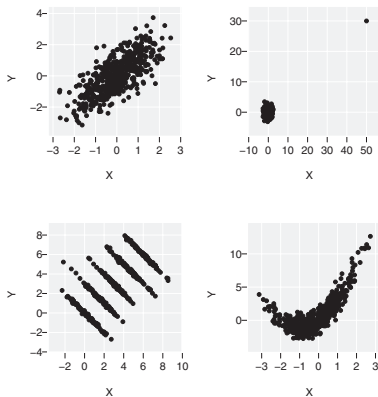
- Because the correlation coefficient is based on standardized variables, it does not depend on the units of measurement
- Thus, the correlation between father's and son's heights would be 0.5 even if the father's height was measured in inches and the son's in centimeters
- Furthermore, the correlation between  $x$  and  $y$  is the same as the correlation between  $y$  and  $x$

## Interpreting the correlation coefficient

- The correlation between heights of identical twins is around 0.93
- The correlation between income and education in the United States is about 0.44
- The correlation between a woman's education and the number of children she has is about -0.2
- When concrete physical laws determine the relationship between two variables, their correlation can exceed 0.9
- In the social sciences, this is rare – correlations of 0.3 to 0.7 are considered quite strong in these fields

## Numerical summaries can be misleading!

From Cook & Swayne's *Interactive and Dynamic Graphics for Data Analysis*:



**Fig. 6.1.** Studying dependence between X and Y. All four pairs of variables have correlation approximately equal to 0.7, but they all have very different patterns. Only the top left plot shows two variables matching a dependence modeled by correlation.

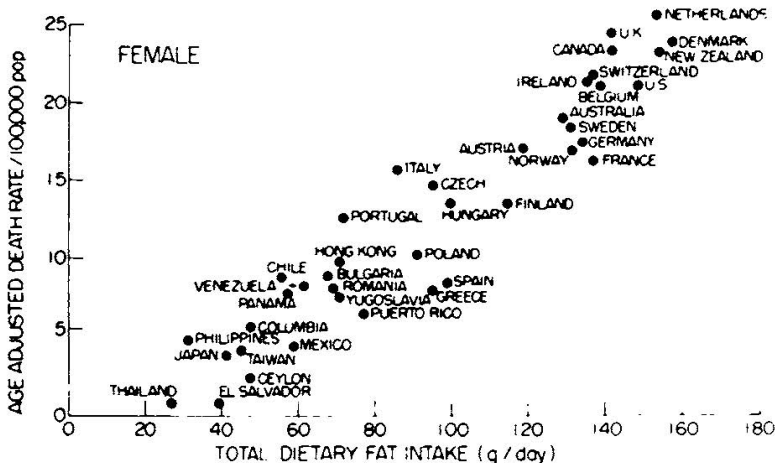


## Ecological correlations

- Epidemiologists often look at the correlation between two variables at the ecological level – say, the correlation between cigarette consumption and lung cancer deaths per capita
- However, people smoke and get cancer, not countries
- These correlations have the potential to be misleading
- The reason is that by replacing individual measurements by the averages, you eliminate a lot of the variability that is present at the individual level and obtain a higher correlation than there really is

## Fat in the diet and cancer

From an article by Carroll in *Cancer Research* (1975):



## Summary

- The standard way to display the relationship between two continuous variables is the scatter plot
- A standard summary statistic for this relationship is the correlation coefficient
- A standardized variable tells us how many standard deviations above/below the mean an observation is:

$$z_i = \frac{x_i - \bar{x}}{SD_x}$$

- Correlations at the ecological level are much higher than correlations at the individual level