Lab 9: z-tests and t-tests

March 21-22, 2017

In lab # 9, we will be covering hypothesis testing using the normal distribution, both by hand and in R.

Note: A hat on a Greek letter indicates an estimator. Our estimator for μ is $\bar{x}$, so when you see $\hat{\mu}$, this is the same thing as $\bar{x}$.

Example 1: A z-test

The distribution of weights for the population of males in the United States is approximately normal with standard deviation $\sigma = 29.8$ pounds. Suppose we believe the mean $\mu = 172.2$.
We conduct an experiment with an n of 50 and find our sample mean to be 180. Conduct a hypothesis test to determine if the true mean is 172.2 based on our data.

$H_0 : \mu = 172.2$

$H_A : \mu \neq 172.2$

$\mu = 172.2$
$\sigma = 29.8$
$\hat{\mu} = 180$
$n = 50$

$z = \frac{\hat{\mu}-\mu}{\sigma/\sqrt{n}}$
$z = \frac{180-172.2}{29.8/\sqrt{50}}$
$z = 1.85$
Find 2-tailed probability using z-table
$p = 2(.0322) = .0644$

Interpret: There is borderline (but not significant) evidence to suggest that the true mean weight of males in the United States may be greater than 172.2, based on this data (p = .0644).

Using R:

```
round(2*pnorm(180,mean=172.2,sd=29.8/sqrt(50),lower.tail=FALSE),4)
```

```
## [1] 0.0642
```

```
round(2*pnorm(1.85,mean=0,sd=1,lower.tail=FALSE),4)
```

```
## [1] 0.0643
```

Creating a confidence interval (z)

Suppose we want to create a 95% confidence interval for μ.

$\hat{\mu} = 180$
$\sigma = 29.8$
$n = 50$ $SE = 29.8/\sqrt(50)$ $z_{\alpha/2} = 1.96$ (from table)

$\hat{\mu} \pm z_{\alpha/2} * SE$
$180 \pm 1.96 * 29.8/\sqrt(50)$
(171.7, 188.3)

Interpret: We can say with 95% confidence that this interval contains the true mean weight of males in the US.

Using R:

```
180+qnorm(.025)*29.8/sqrt(50)
```

```
## [1] 171.74
```

```
180+qnorm(.975)*29.8/sqrt(50)
```

```
## [1] 188.26
```

```
# In one step (using a vector)
180+qnorm(c(.025,.975))*29.8/sqrt(50)
```

```
## [1] 171.74 188.26
```

Example 2: A t-test

Let's consider the same situation as before, but now suppose we don't know the standard deviation σ.

The distribution of weights for the population of males in the United States is approximately normal. Suppose we believe the mean $\mu = 172.2$.
We conduct an experiment with an n of 50 and find our sample mean to be 180 and sample standard deviation to be 30. Conduct a hypothesis test to determine if the true mean is 172.2 based on our data.

$H_0 : \mu = 172.2$

$H_A : \mu \neq 172.2$

$\mu = 172.2$
$\hat{\mu} = 180$
$s = 30$
$n = 50$
$df = n - 1 = 49$

$t = \frac{\hat{\mu}-\mu}{s/\sqrt{n}}$
$t = \frac{180-172.2}{30/\sqrt{50}}$
$t = 1.84$
Find 2-tailed probability using t-table
$.05 < p < .1$

Interpret: There is borderline (but not significant) evidence to suggest that the true mean weight of males in the United States may be greater than 172.2, based on this data (.05 < p < .1).

Using R:

```
2*pt(1.84,df=49,lower.tail=FALSE)
```

```
## [1] 0.07182936
```

Constructing a confidence interval (t)

Suppose we want to create a 95% confidence interval for μ.

$\hat{\mu} = 180$
$s = 30$
$n = 50$ $SE = 30/\sqrt(50)$ $t_{\alpha/2} = 2.01$ (from table)

$\hat{\mu} \pm z_{\alpha/2} * SE$
$180 \pm 2.01 * 30/\sqrt(50)$
$(171.4, 188.5)$

Interpret: We can say with 95% confidence that this interval contains the true mean weight of males in the US.

Using R:

```
180+qt(c(.025,.975),49)*30/sqrt(50)
```

```
## [1] 171.4741 188.5259
```

Notice that this is a little wider than the z-interval, since we estimated the standard error. This becomes more pronounced at small sample sizes.

Practice

Number one

Suppose that in a particular geographic region, the mean and standard deviation of scores on a reading test are 100 points, and 12 points, respectively. Our interest is in the scores of 55 students in a particular school who received a mean score of 96. We can ask whether this mean score is significantly lower than the regional mean-that is, are the students in this school comparable to a simple random sample of 55 students from the region as a whole, or are their scores surprisingly low? Also calculate a 95% confidence interval based on this sample. (from Wikipedia)

Number two

Suppose that the average IQ is 100; perform a test to see if the children in the lead-IQ dataset are average. Create a confidence interval for the mean IQ based on this data.

Answers

```r
# 1
2*pnorm(96,100,12/sqrt(55))
96+qnorm(c(.025,.975))*12/sqrt(55)

# 2
leadIQ<-read.delim("http://myweb.uiowa.edu/pbreheny/data/lead-iq.txt")
mu.hat<-mean(leadIQ$IQ)
s<-sd(leadIQ$IQ)
n<-length(leadIQ$IQ)
t<-(mu.hat-100)/(s/sqrt(n))
2*pt(t,n-1)

mu.hat+qt(c(.025,.975),n-1)*s/sqrt(n)

# Using t-test function (good for when given data)

t.test(leadIQ$IQ,mu=100,alternative="two.sided")
```

```
## [1] 0.01343347
## [1] 92.82862 99.17138
## [1] 2.486475e-10
## [1] 88.52022 93.64107
##
##  One Sample t-test
##
## data:  leadIQ$IQ
## t = -6.8955, df = 123, p-value = 2.486e-10
## alternative hypothesis: true mean is not equal to 100
## 95 percent confidence interval:
##  88.52022 93.64107
## sample estimates:
## mean of x
##  91.08065
```

Note: Although the average IQ in this dataset is significantly lower than 100, “average” IQ is defined as between 90 and 110, and the mean in this group was 91.

Interpretation note: Remember that when we say “95% confidence” about an interval, this does NOT mean that there is a 95% probability of the true mean being in the interval. It means that if we were to repeat this experiment a lot of times, 95% of the intevals constructed in this manner would contain the true mean. It’s a bit of a touchy subject, so overall just be careful to not say “probability” when you’re interpreting confidence intervals.