6 Binomial practice problems

1. Suppose a group of 20 men, all unrelated, received a flu vaccine. Assume each man in this group has a 0.05 chance of dying in the next year.

   How likely it is that at least 2 of these men will die in the following year?

$$\begin{aligned} P[X \geq 2] &= 1 - P[X < 2] \\ &= 1 - (P[X = 0] + P[X = 1]) \\ &= \texttt{1 - sum(dbinom(0:1, size = 20, prob = 0.05))} \\ &= \boxed{0.264} \end{aligned}$$

2. Suppose 67% of Americans watch TV on a daily basis. Suppose repeated samples of size 19 are drawn from the U.S. population.

   What is the probability that at least 3 of the randomly selected individuals watch TV on a daily basis?

$$\begin{aligned} P[X \geq 3] &= 1 - (P[X = 0] + P[X = 1] + P[X = 2]) \\ &= 1 - \left(\binom{19}{0} (0.67)^0 (0.33)^{19-0} + \binom{19}{1} (0.67)^1 (0.33)^{19-1} + \binom{19}{2} (0.67)^0 (0.33)^{19-2} \right) \\ &= \boxed{0.9999995} \end{aligned}$$

7 Normal practice problems

1. Find the area under the normal curve...

   (a) below 0.3.

$$\begin{aligned} P[X \leq 0.3] &= \texttt{pnorm(.3)} \\ &= \boxed{0.6179114} \end{aligned}$$

   Using the table, we find the probability to be 0.618.

   (b) above 0.65.

$$\begin{aligned} P[X \geq 0.65] &= \texttt{1 - pnorm(0.65)} \\ &= \boxed{0.2578461} \end{aligned}$$

$$OR$$

$$\begin{aligned} P[X \geq 0.65] &= \texttt{pnorm(-0.65)} \\ &= \boxed{0.2578461} \end{aligned}$$

   Using the table, we find the probability to be $1 - 0.742 = 0.258$.

   (c) between 0.3 and 0.65.

$$\begin{aligned} P[0.3 \leq X \leq 0.65] &= \texttt{pnorm(0.65) - pnorm(0.3)} \\ &= \boxed{0.1242425} \end{aligned}$$

$$OR$$

$$\begin{aligned} P[0.3 \leq X \leq 0.65] &= \texttt{1 - (pnorm(0.3) + pnorm(-0.65))} \\ &= \boxed{0.1242425} \end{aligned}$$

   Using the table, we find the probability to be $0.742 - 0.618 = 0.124$.

(d) below -0.45.

$$P[X \leq -0.45] = \texttt{pnorm(-0.45)}$$
$$= \boxed{0.3263552}$$

Using the table, we find the probability to be 0.326.

2. Find the following percentiles of the normal curve.

(a) 20^{th}

$$P[Z \leq z] = 0.1$$
$$z = \texttt{qnorm(0.2)}$$
$$= \boxed{-0.8416212}$$

Using the table, we find the percentile to be -0.84.

(b) 80^{th}

$$P[Z \leq z] = 0.80$$
$$z = \texttt{qnorm(0.80)}$$
$$= \boxed{0.8416212}$$

Using the table, we find the percentile to be 0.84.

(c) 95^{th}

$$P[Z \leq z] = 0.95$$
$$z = \texttt{qnorm(.95)}$$
$$= \boxed{1.644854}$$

Using the table, we find that the percentile is between 1.64 and 1.65. From the `R` code above, we see that the percentile is actually rounded to 1.645; this is value commonly used for the 95^{th} percentile.

(d) 90^{th}

$$P[Z \leq z] = 0.90$$
$$z = \texttt{qnorm(.90)}$$
$$= \boxed{1.281552}$$

Using the table, we find the percentile to be 1.28.

8 Categorical practice problems

1. Use the table below summarizing the survival data at gestational age 22 weeks to answer the following questions.

Outcome	Count
Survived	0
Died	29

(a) What are the exact 95% Confidence Limits for probability of surviving?

$$\begin{aligned}\text{Lower Bound} &= 1 - 95\% \text{ Upper Conf Limit} \\ &= 1 - 1.0000 \\ &= \boxed{0.0000} \\ \text{Upper Bound} &= 1 - 95\% \text{ Lower Conf Limit} \\ &= 1 - 0.8806 \\ &= \boxed{0.1194}\end{aligned}$$

(b) What is the p-value for the approximate test and exact test?

$$\begin{aligned}\text{approx} &= \boxed{< .0001} \\ \text{exact} &= \boxed{3.725\text{E-}09}\end{aligned}$$

(c) What test does the p-value correspond to?

$$\begin{aligned}H_0 &: p = 0.50 \\ &vs. \\ H_1 &: p \neq 0.50\end{aligned}$$

2. Use the smoking data set to answer the following questions.

(a) What proportion of the observations survived?

$$\frac{\#\text{survived}}{\#\text{observations}} = \boxed{0.7191781}$$

(b) What is the exact confidence interval for survival?

$$\text{CI} = \boxed{(0.6940270,\ 7433448)}$$

(c) What is the exact p-value testing that the proportion of survival is equal to 0.5?

$$P[p \geq 0.7191781] = \boxed{< 2.2e-16}$$