Math 6000, Fall 2020 (Prof. Kinser), Study Checks

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1. There exists unique morphism $x \to y$ iff $x \leq y$ in P (this defines a composition).

Check category axioms.

Definitions and Theorems

A *partial order* (POSET) on a nonempty set A is a relation $\leq$ on A satisfying

(1) $x \leq x$ for all $x \in A$ (reflexive)

(2) if $x \leq y$ and $y \leq x$ then $x = y$ for all $x, y \in A$ (antisymmetric).

(3) if $x \leq y$ and $y \leq z$ then $x \leq z$ for all $x, y, z \in A$ (transitive).

From the notation above, the **objects** are sets (set A).

The morphism is as follows: $x \to y$ iff $x \leq y$ in A.

Composition Law:

$x \leq y$ and $y \leq z \Rightarrow x \leq z$ (by transitivity).

satisfying:

(i) If $x \to y(\iff x \leq y)$ and $a \to b(\iff a \leq b)$. Then they are are disjoint unless $a = x$ and $b = y$.

(ii) Associativity: Suppose $h : a \to b, g : b \to c, f : c \to d)$.

$a \leq b, b \leq c, c \leq d$. Then,

$h(gf) \Rightarrow a \leq (b \leq d) \iff a \leq d$ and $(hg)f \iff (a \leq c) \leq d \iff a \leq d$. Check

(iii) identity morphism: $x \leq x(x \to x)$. -¿ reflexive

SE 2. Modify Proof of $(\Rightarrow)$ for category of groups (pg. 10 notes)

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3. Think of a counterexample (isomorphism, not same as retraction and section)

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4. Check axioms of Power Set for functor (notes pg. 12) . Same for example 3/4 maybe (pg. 13)

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5. Check Functor **Groups** $\to$ **Groups** have things like $G \mapsto Z(G)$.

SE 6. (Natural Transformations) $F, F' : \mathcal{C} \to \mathcal{D}$ makes $Fun(\mathcal{C}, \mathcal{D})$ a category.

SE 7. Suppose A free on X, B free on Y in the same category $\mathcal{C}$. Let $\phi : X \xrightarrow{\sim} Y$ be a bijection.

Use **universal property** to show $\exists! A \xrightarrow{f:\sim} B$ (iso) in $\mathcal{C}$ such that diagram commutes (pg. 23 in notes)

SE 8. Trivial set is a free-object.

SE 9. Let I, I' be initial objects in $\mathcal{C}$. Prove that there exists a **unique** isomorphims $f : I \to I'$

("initial objects are unique" upto isomorphism."

10. (**Unimportant Study Exercise** Let $\mathcal{C} =$ **Sets**, $\mathcal{D}$ concrete category (ex: groups) and $F : \mathcal{D} \to \mathcal{C}$, the forget functor.

Prove that an initial object in the category $(X \downarrow F)$ (where X is any set) is the same thing as **free objects in $\mathcal{D}$ on X.**

11. **Review:** Submodule, Quotient modules, isomorphism theorem, and homomorphism.

SE 12. **Proposition:** Let $(M, +)$ be an abelian group, then M is the structure of $R-$module is equivalent to satisfying a ring homomorphism:

$$\phi : R \to End_{\mathbb{Z}}(M)$$

Check Details

13. **(Unimportant Study Exercise)** This is equivalent to giving ring homomorphism $S \to End_R(M)^{op}$ or $R \to End_S(M)$.

14. **Unimportant Study Exercise** (Construct nonstandard $\mathcal{C} - \mathcal{C}$ bimodule structure on $\mathcal{C}$.

SE 15. Direct Sum is Co-Product, Product is product in categorical sense (go over universal property stuff).

SE 16. Repeat with **Coproduct** (on pg. 41 in notes)

SE 17. eR and $(1-e)R$ are 2-sided ideals of R and $R = eR \bigoplus (1-e)R$.

SE 18. Free-modules are free objects in the category of $R-$ objects.

19. Repeat Example 3 in detail like class both **computationally** and **symbolically** when both are bi-modules.