2022 Iowa PDE Conference
Title and Abstracts

1 Invited Talks

A Liouville theorem for a class of fully non-linear equations
Hao Fang, The University of Iowa

Abstract: In this talk, we study in Euclidean 4-space a class of fully non-linear PDEs. It is closely related to classical Hessian type equation and the uniformization problem in conformal geometry. We discuss the background and motivation of our study and present our key approach, which include some standard estimate, a key monotonicity formula used in our previous works and careful study of the geometry of level sets. We will also discuss some of the geometrical applications, which includes a Penrose type inequality for certain asymptotically hyperbolic spaces. This is joint work with Biao Ma of PKU and Wei Wei of Nanjing University.

Global dynamics and photon loss in the Kompaneets equation
Hailiang Liu, Iowa State University

Abstract: The Kompaneets equation governs dynamics of the photon energy spectrum in certain high temperature (or low density) plasmas. We prove several results concerning the long-time convergence of solutions to Bose–Einstein equilibria and the failure of photon conservation. In particular, we show the total photon number can decrease with time via an out flux of photons at the zero-energy boundary. The ensuing accumulation of photons at zero energy is analogous to Bose–Einstein condensation. We provide two conditions that guarantee that photon loss occurs, and show that once loss is initiated then it persists forever. We prove that as time tends to infinity, solutions necessarily converge to equilibrium and we characterize the limit in terms of the total photon loss. Additionally, we provide a few results concerning the behavior of the solution near the zero-energy boundary, an Oleinik inequality, a comparison principle,and show that the solution operator is a contraction in L1. None of these results impose a boundary condition at the zero-energy boundary. This is a joint work with J. Ballew, G. Iyer, D. Levermore and R. Pego.

Some recent results in two species competition
Rana Parshad, Iowa State University

Abstract: We investigate certain two species ODE and PDE Lotka–Volterra competition models, where one of the competitors could potentially go extinct in finite time. We show that in this setting, various novel dynamics are possible. In particular competitive exclusion can be avoided, and the slower diffusing competitor may not win. Extensions are made to the ”very” fast diffusion case, wherein novel dynamics emerge as well. Numerical simulations are performed to corroborate our analytical findings.

Gradient estimates for nonlinear elliptic double obstacle problems
Seungjin Ryu, The University of Iowa/University of Seoul

Abstract: In this talk, we study the regularity estimates for nonlinear ellipitic double obstacle problems. The partial differential operator is of the p-Laplacian type and includes merely measurable coefficients in one variable. We discuss that the gradient of a weak solution is as integrable as both the gradient of assigned two obstacles and the nonhomogeneous divergence term under a small BMO semi-norm assumption on the coefficients in the other variables.

Complex Bursting Patterns in Respiratory Neurons
Yangyang Wang, The University of Iowa

Abstract: Pre-Bötzinger complex (pre-BötC) network activity within the mammalian brainstem controls the inspiratory phase of the respiratory rhythm. Experimental recordings in brainstem slices containing pre-BötC have revealed the existence of a variety of complex bursting activity patterns depending on distinct combinations of burst-generating INaP and ICAN conductances. Respiratory neuron systems exhibit separated timescales and can be challenging to study since the associated equations are singularly perturbed. In this work, we use multiple timescale decomposition, bifurcation analysis and computational modeling to uncover mechanisms underlying several different types of intrinsic bursting dynamics observed experimentally in pre-BötC neurons. Our analysis also yields predictions about how changes in the balance of the two bursting mechanisms contribute to alterations in inspiratory pacemaker neuron activity during prenatal development.

Capillary Gravity Water Waves Linearized at Monotone Shear Flows: Eigenvalues and Inviscid Damping
Chongchun Zeng, Georgia Institute of Technology

Abstract: We consider the 2-dim capillary gravity water wave problem – the free boundary problem of the Euler equation with gravity and surface tension – of finite depth $x_2 \in (-h, 0)$ linearized at a uniformly monotonic shear flow $U(x_2)$. Our main focuses are eigenvalue distribution and inviscid damping. We first prove that in contrast to finite channel flow and gravity waves, the linearized capillary gravity wave has two unbounded branches of eigenvalues for high wave numbers. They may bifurcate into unstable eigenvalues through a rather degenerate bifurcation. Under certain conditions, we provide a complete picture of the eigenvalue distribution. Assuming there are no singular modes (i.e. embedded eigenvalues), we obtain the linear inviscid damping. We also identify the leading asymptotic terms of the velocity and obtain faster decay for the remainders. This is a joint work with Xiao Liu.

2 Student/Post-doc Session

José David Beltrán Lizarazo, The University of Iowa

In this proposal we prove the existence of a global Lipschitz-continuous solutions for the Cauchy problem associated to a linearly damped p-system of the form

$$\begin{cases} v_t - u_x = 0 & (x,t) \in \mathbb{R} \times (0,\infty) \\ u_t + p(v)_x = -\alpha u, \end{cases} \tag{1}$$

where p is a smooth function in (b,∞) for $b>0$ and satisfies

$$p'(v) < 0, \ p''(v) > 0, \ \lim_{x \longrightarrow b^+} p(v) = \infty.$$

With bounded and measurable initial data

$$(v(x,0), u(x,0)) = (v_0(x), u_0(x)). \tag{2}$$

and $v_0(x) \geq \delta > 0$.
Systems of this form are usually used as models for the dynamics of compressible gases through porous media, where v denotes the specific volume of the gas, u denotes its velocity and p its pressure.

We face this problem through a variant of the well-known vanishing viscosity method; first, with the aid of Riemann Invariants globally defined for the system, we transform the problem into a diagonal quasi-linear form:

$$\begin{cases} r_t + \lambda r_x = -\alpha \left(\frac{r+s}{2}\right) \\ s_t + \mu s_x = -\alpha \left(\frac{r+s}{2}\right) \end{cases}$$

with initial data

$$(r(x,0), s(x,0)) = (r_0(x), s_0(x)).$$

Then, we consider parabolic perturbations of this system of the form

$$\begin{cases} r_t + \lambda r_x = \epsilon r_{xx} - \alpha \left(\frac{r+s}{2}\right) \\ s_t + \mu s_x = \epsilon s_{xx} - \alpha \left(\frac{r+s}{2}\right) \end{cases}$$

The local existence of smooth solutions for the above problem is obtained using tools from the theory of parabolic equations and additional estimates over the solution and its derivatives allow us to extend these solutions globally.
Once we have obtained a sequence of solutions $\{w^\epsilon\}$ (with $w^\epsilon = (v^\epsilon, u^\epsilon)$) we study the behaviour of w^ϵ when $\epsilon \longrightarrow 0$. The theory of compensated compactness and its application to strictly hyperbolic systems support us to find a subsequence $\{w^\epsilon\}$ that converges in the weak $\star$ topology of $L^\infty(\mathbb{R} \times [0,\infty))$ to a weak solution of the original problem;

$$w^\epsilon \overset{*}{\rightharpoonup} w.$$

Finally, additional uniform estimates over the space and time derivatives of our sequence of solutions lead us to show that the sequence $\{w^\epsilon\}$ is actually contained in a Sobolev space $W^{1,\infty}(\mathbb{R} \times (0,T))$ for a finite time T. This, together with the classical embedding theorems, prove that the sequence converges uniformly on bounded sets to a pair of functions (v,u) that are a weak global Lipschitz continuous solution of the original problem.

Nonlocal equations on the boundary
Mitchell Haeuser, Iowa State University

Abstract: We will discuss regularity for a problem involving a fractional Dirichlet-to-Neumann operator associated to harmonic functions. In particular, we will define a fractional power of the normal derivative, compatible Sobolev spaces, and consider various examples. We will further look at the extension problem characterization to obtain various estimates. This is joint work with Luis Caffarelli (UT Austin) and Pablo Raúl Stinga (Iowa State University)

Weak-strong uniqueness for Maxwell–Stefan systems
Xiaokai Huo, Iowa State University

Abstract: The weak-strong uniqueness for Maxwell–Stefan systems and some generalized systems is proved. The corresponding parabolic cross-diffusion equations are considered in a bounded domain with no-flux boundary conditions. The key points of the proofs are various inequalities for the relative entropy associated with the systems and the analysis of the spectrum of a quadratic form capturing the frictional dissipation. The latter task is complicated by the singular nature of the diffusion matrix. This difficulty is addressed by proving its positive definiteness on a subspace and using the Bott–Duffin matrix inverse. The generalized Maxwell–Stefan systems are shown to cover several known cross-diffusion systems for the description of tumor growth and physical vapor deposition processes.

Well-Posedness of BV Solutions to a System of Balance Laws Arising in Traffic Flow
Nitesh Mathur, The University of Iowa

Abstract: The goal of this talk is to outline conditions that are needed to exhibit well-posedness and asymptotic behavior of BV solutions to a system of balance laws modeling traffic flow. We proved the results by finding entropy-entropy flux pair and verifying Kawashima condition, sub-characteristic condition, and the partial dissipative inequality in the framework of Dafermos.

Compactness Method for Boundary Regularity of the Navier-Stokes Equation
Bryanna R. Petentler, The University of Iowa

Abstract: In this talk, we look at suitable weak solutions of the Navier-Stokes equation in three dimensions and investigate the Hölder regularity of these solutions up to the boundary. The Hölder regularity up to flat boundaries was already proven by Seregin in 2002. We prove boundary regularity result with a compactness argument and monotonicity properties of harmonic functions.

The effect of "fear" in two species competition
Vaibhava Srivastava, Iowa State University

Abstract: Non-consumptive effects, such as fear, can strongly influence predator-prey dynamics. These effects have not been well studied in the case of competitive systems, despite ecological and social motivations for the same. In this work, we consider the effect of fear on the classic two-species Lokta-Volterra competition model. We consider both the ODE setting and the PDE setting, where fear can be inhomogeneous in space. We find that the effect of fear can have counter-intuitive effects on the classical scenarios of weak and strong competition and competitive exclusion.