Ranked-Choice Voting and Brexit

Methods and Outcomes

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Abstract

At the time of writing, the United Kingdom has yet to reach a consensus on a plan to leave the European Union. Here we explore, given a few assumptions, whether a single-winner ranked-choice vote could have permitted an alternate outcome: the UK withdrawing from the EU with a plan for the future.

Contents

1 Introduction

1.1 Definitions

To help better understand the contents of this report, it is helpful to lay out a few definitions.

Definition (Ballot)**.** A ballot is a vote cast in a ranked-choice voting election. For example, suppose we have an election where three candidates are competing for one seat; call these candidates C_1, C_2, and C_3. Then, a sample ballot for this election would look something like

$$[C_2,\ C_1,\ C_3],$$

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where the left-to-right ordering of the candidates indicates the voter's preference (from most preferred to least preferred). In this case, the voter prefers C_2 the most, C_1 second-most, and C_3 least.

Definition (Exhaustible or Exhausted Ballot)**.** An exhaustible ballot is one where the number of candidates ranked on the ballot does not match the number of candidates running in the election. As in the previous definition, consider the same three candidates competing for one seat. An exhaustible ballot would look like

$$[C_3, ?, ?],$$

where the second and third ballot slots do not specify a candidate. Thus, an exhausted ballot is one that, once the last candidate listed is eliminated, no longer counts toward final vote totals. In our example, if C_3 is eliminated, this ballot becomes exhausted and is not redistributed to another candidate.

Definition (Brexit Plan)**.** In this report, a plan refers to one of the proposed Brexit plans. This set of plans is detailed further in A.1. Furthermore, in the context of this report, the "candidates" in all ranked-choice elections are the Brexit plans themselves; i.e. we are attempting to simulate Parliament conducting an RCV election to select a Brexit plan.

Definition (Positive Vote, Shared Vote)**.** A positive vote by a party is a vote in which more than 50% of a party's MPs vote "Aye" on a motion. A shared vote is a vote on which a particular MP and a particular MP have voted "Aye."

1.2 Data Collection

All voting data were collected from CommonsVotes, the UK House of Commons' public-facing vote record system [**?**]. As opposed to the method used for estimating plan preferences for an individual Member of Parliament (MP), for party-line rankings, we only consider votes on the Brexit plans themselves; these votes are shown below.

Motion	Date	Vote Number	CommonsVotes
B, H, K, L, O	March 27th, 2019	386, 388, 390, 391, 393	655, 657, 659, 660, 662
C, D, E, G	April 1st, 2019	397-400	666-669

Additionally, for party-line votes, we operate on the principle of most recent information; that is, for motions voted on twice, *we only consider the results of the most recent vote on that motion*.

In order to better estimate the particular preferences of individual MPs, we expanded our dataset by analyzing divisions, or Parliamentary recorded floor votes, from June 28th, 2017 to June 12th, 2019. This represents 416 total divisions' worth of data.

1.3 Counting Ballots

In this experiment, we conduct an election using a single-winner ranked choice voting system. In doing so, we follow the below procedure to elect a plan:

1. All plans are awarded the number of ballots on which they are ranked first.

2. The plan with the fewest number of awarded ballots is eliminated; if more than one plan is tied for the fewest number of ballots, randomly choose one plan to eliminate. Then, for each ballot previously awarded to that plan, the next-highest-ranked plan on the ballot is awarded the ballot. In other words, the eliminated plan's ballots are redistributed to the other plans by order of preference. In the case that a ballot is exhaustible and the plan eliminated is the last-ranked plan on that ballot, we do not redistribute that ballot to a remaining plan.

3. Repeat (1) and (2) until there is one plan remaining. The remaining plan is the winner.

2 Ranking Plans

2.1 Party Lines

To simulate a party-line vote, we create a party-block ballot – that is, a collection of ballots in which each ballot is affiliated with the same party, and each ballot identically ranks plans. For example, consider the Labour Party, which has 243 members. Out of all cast ballots, there will be at least 243 with identical rankings.

2.1.1 Procedure

How do we get from a voting record without ranked-choice data to the rankings outlined in 3.1.1? Consider the following mock election. Here's a a sample of the voting record for plan P_1:

Plan	Party	Vote
P_1	Interplanetary	Aye
P_1	Interplanetary	No
P_1	Interplanetary	Aye
P_1	Interplanetary	No
P_1	Interplanetary	Aye
P_1	Hooky	No Vote Recorded
...	...	...

We can see that, within the Interplanetary party, members vote 3-2 in favor of plan P_1. Thus, we conclude that the party prefers plan P_1 with 60% support.

Now, suppose we collect the same data on plans P_1 through P_4 (not shown); we then find that the Interplanetary party ranks plans in the following way:

Party	Plan	Support	Rank
Interplanetary	P_3	100%	1
Interplanetary	P_1	60%	2
Interplanetary	P_2	40%	3
Interplanetary	P_4	0%	4

Now, if we perform the same procedure for each party, we can come up with a set of party-line rankings:

Party	Members	Ranking
Interplanetary	5	$[P_3, P_1, P_2, P_4]$
Hooky	26	$[P_1, P_2, P_3, P_4]$
Anti-Leadership	11	$[P_4, P_3, P_2, P_1]$

Thus, our election would have five ballots with ranking $[P_3, P_1, P_2, P_4]$, 26 with ranking $[P_1, P_2, P_3, P_4]$, and 11 with ranking $[P_4, P_3, P_2, P_1]$; these groups of ballots are our party-block ballots. We follow this same procedure to generate party-block ballots for Brexit plans.

2.2 Individuals

To provide more granularity to the experiment, we take on a more rigorous task: estimating how individual MPs would rank Brexit plans. To do so, we find out how "close" or "far" individual MPs are from each party regardless of their official party membership. Then, we adjust the MP's Brexit plan preferences according to their closeness to each party, alongside additional weights we outline in B.1.1.

2.2.1 Probabilistic Weighting

Our first task is to weight each party probabilistically [1]with respect to each MP.

At their cores, our measures of closeness rely on the set of votes shared by an MP and a party. Suppose we are looking at the relationship between a specific MP and a party. Let the set of votes on which the MP voted "Aye" be labeled m and the set of votes on which a majority of the party's MPs – typically 50% – vote "Aye" be labeled A.

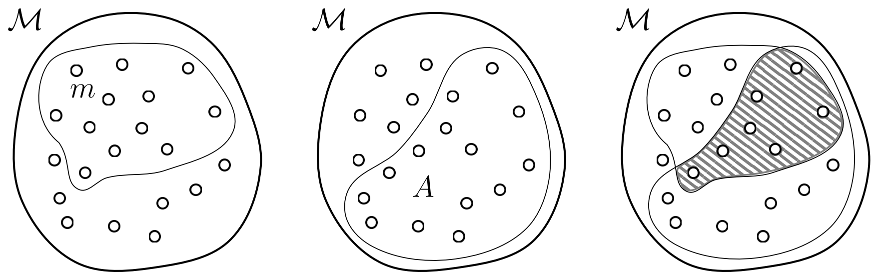
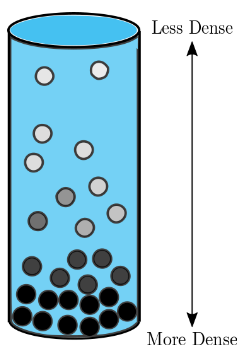


Figure 1: *Here, we can see that m (our MP's votes) and A (our party's votes) are both subsets of $\mathcal{M}$, our set of motions. The figure on the far right shows the intersection of m and A; this represents the set of motions on which m voted "Aye" and A had a majority of its members vote "Aye."*

Then, we say that the set of shared votes (i.e. the intersection of m and A) determines how "close" m and A are. In other words, if an MP and a party vote with each other often, they are close.

We also use two additional probabilistic weighting methods – conditional and ratio weights – outlined in B.1.3 and B.1.4. These two weighting methods attempt to rectify the shortcomings of the intersection weighting method as detailed in B.1.2.

2.2.2 Procedure



Now we can move on to the way weights are assigned. In order to do so, think about the way particles settle in water: denser particles settle toward the bottom, and those less dense always settle toward the top. The order of the individual particles might not always be exactly the same every time you shake up the water, but the results will always be similar, as more dense particles tend toward the bottom and less dense tend toward the top. Now, think about each Brexit plan as one of these particles. If we can figure out a way to assign each particle a density, we can figure out, for each MP, which of the plans are "dense" (well-liked) and which are "light" (disliked). How, though, do we go about assigning these densities?

[1] To justify our use of the word "probabilistic," it is important to show that the set of motions $\mathcal{M}$ is, in fact, a sample space. Firstly, $\mathcal{M}$ is mutually exclusive, as each motion is unique. Secondly, if MPs and parties are treated as events (as in 1), all MPs and parties are contained entirely within $\mathcal{M}$. Thus, $\mathcal{M}$ is a sample space.

Let us look at one MP of the Interplanetary party; call this MP M. After a bit of digging through the voting record, we find out that M has the following weights, according to their voting tendencies:

Party	Intersection	Conditional	Ratio
Interplanetary	0.94	0.923	9.03
Hooky	0.86	0.83	4.03
Anti-Leadership	0.307	0.3	0.28

Clearly, M is quite close to the Interplanetary party, but also votes with the Hooky party much more often than not. The voting relationship with the Anti-Leadership party is tenuous at best, and these weights are emblematic of the inherent issue with the intersection weighting method, as outlined in B.1.2. Now, recall the preference rankings for each of the parties, with the support scores for each plan adjoined:

Party	Ranking	Support Scores (%)
Interplanetary	$[P_3, P_1, P_2, P_4]$	$[100, 60, 40, 0]$
Hooky	$[P_1, P_2, P_3, P_4]$	$[96, 77, 54, 12]$
Anti-Leadership	$[P_4, P_3, P_2, P_1]$	$[100, 82, 55, 18]$

The density assignment algorithm is reasonably simple: each plan is given a "density" according to how much it is preferred by each MP and each party. Since M is close to the Interplanetary Party and since the Interplanetary Party likes plan P_3, we can say that, for M, P_3 is dense. P_4, on the other hand, is light, as it is not supported at all by the Interplanetary Party. Below are the remaining densities for M, outlining M's relationship to the Interplanetary Party:

Plan	Density = (support score $\times$ party weight)
P_1	$0.564 = 0.6 \times 0.94$
P_2	$0.376 = 0.4 \times 0.94$
P_3	$0.94 = 1 \times 0.94$
P_4	$0 = 0 \times 0.94$

The closer an MP is to a party (high party weight) and the more that party likes a plan (high support score), the greater density a plan has.

Essentially, the closer an MP is to a party (indicated by a high party weight) and the more that party likes a plan (indicated by a high support score), the greater density that plan has. If we continue this procedure – examining M's relationships to each party – we end up with the following densities for each plan:

Plan	Density = sum of (support score $\times$ party weight) for each party
P_1	$1.44486 = (0.6 \times 0.94) + (0.96 \times 0.86) + (0.18 \times 0.307)$
P_2	$1.20705 = (0.4 \times 0.94) + (0.77 \times 0.86) + (0.55 \times 0.307)$
P_3	$1.65614 = (1 \times 0.94) + (0.54 \times 0.86) + (0.82 \times 0.307)$
P_4	$0.41020 = (0 \times 0.94) + (0.12 \times 0.86) + (1 \times 0.307)$

Now, we simply order the plans (most to least dense), and this is our MP's preference order. In this case, M has a plan ranking of $[P_3, P_1, P_2, P_4]$. Then, M casts a ballot with this plan ranking. If we perform the same procedure for each MP, we can build a set of ballots that reasonably reflects the voting tendencies of the MPs.

We provide more detail on this algorithm in B.1.6.

2.2.3 Additional Variance

There are also other factors we are able to vary in order to change the weights assigned to each party and plan.

1. We can change our definition of "majority support." By default, we assume that if more than 50% of a party's MPs vote "Aye" on a motion, then the entire party supports that motion. However, this is not in alignment with reality – half a group's support of a plan is irreflective of the entire group's sentiment. What if we consider majority support to be 60% of a party's MPs? 66%? What about higher values? By changing this boundary, we are also shrinking the pool of shared votes – thus, parties who vote as a bloc will have more MP preference weight than parties who often split votes.

2. If we consider a "No Vote Recorded" vote to be tacit support for a motion, we can increase the pool of shared votes.

3. We can restrict the set of divisions from which we extract data; as mentioned in 1.2, we use 416 divisions which have taken place over the last two years. What if we use fewer divisions' worth of data? What if we choose particularly idealogical votes to analyze instead of using all? If we do so, will the number of shared votes in our restricted set of divisions remain proportional to the number of shared votes in the whole set?

3 Results

3.1 Party Lines

3.1.1 Rankings

In table 1, we include the parties' rankings according to the procedure in 2.1.1. Note that the data in the "Preference Order" column is ordered from most preferred to least preferred, based on the ranking scheme outlined in 2.1.1.

Party	Preference Order	No Preference
Conservative	B, O, H, C, D, E, L, G, K	–
Democratic Unionist*	O	B, C, D, E, G, H, K, L
Deputy Speaker†	–	B, C, D, E, G, H, K, L, O
Green*	L, E, G	B, C, D, H, K, O
Independent*	E, L, G, C, D, K, H	B, O
Liberal Democrat*	E, L, G, D, C	B, H, K, O
Labour	K, C, E, D, G, L, H, B, O	–
Plaid Cymru*	L, D, E, G	B, C, H, K, O
Scottish National*	L, D, G, E	B, C, H, K, O
Sinn Féin†	–	B, C, D, E, G, H, K, L, O
Speaker†	–	B, C, D, E, G, H, K, L, O

Table 1: *: *this party's ballots are exhaustible.*
†: *this party does not participate in this election, as they do not rank plans.*

Plan B wins over plan K, as plan B has 312 votes to K's 264. With the implementation of exhausted ballots, plan B wins every simulated election, regardless of sample size. We can see that many of the ballots (62 of them, to be exact) exhaust after plan L is eliminated; this is attributable to the coalition seemingly formed by the Green, Independent, Liberal Democrat, Plaid Cymru, and Scottish National parties, all of whom ranked plans E and L within (at most) their top four plans. Then, as all but the Independents ranked none of the remaining plans (B or K), these ballots exhaust when L is eliminated.

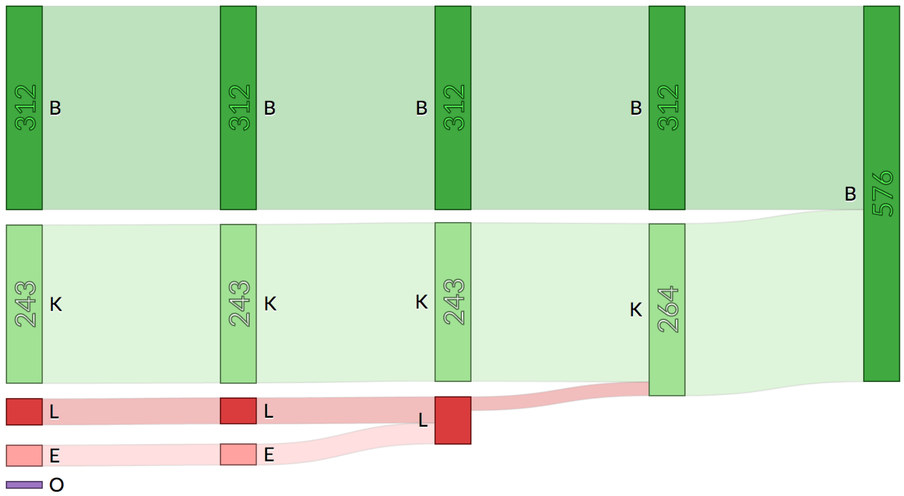


Figure 2: *Vote transfer Sankey diagram for the non-random party-line election. Letters represent plan names.*

Based on this data, we can conclude that if the Brexit plan was selected by a ranked-choice voting system and the MPs of each party voted identically to each other, plan B would be the plan of choice. This is a logical outcome as the Conservative party – champions of plan B – are the largest party.

3.1.2 Simulated Elections with Randomized Ballots

To add a small amount of variance to the party-line vote experiment, we randomize plan preferences for exhaustible ballots; i.e. if we have the $[C_3, ?, ?]$ ballot from 1.1, we randomly choose plans for the two empty ballot slots, then cast the new ballot.

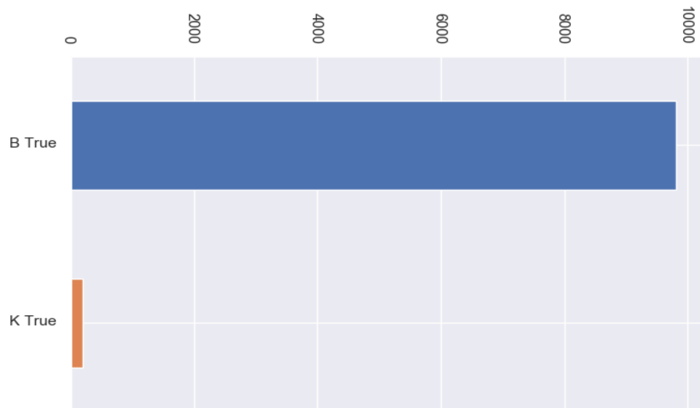


Figure 3: *Proportion of 10,000 simulated elections won by each plan. The truth value next to each plan name indicates whether it was a Droop quota winner.*

In 3, we see that, over 10,000 simulated elections with randomized exhaustible ballots, plan K won approximately two percent (or 200) of 10000 elections, with plan B winning the rest.

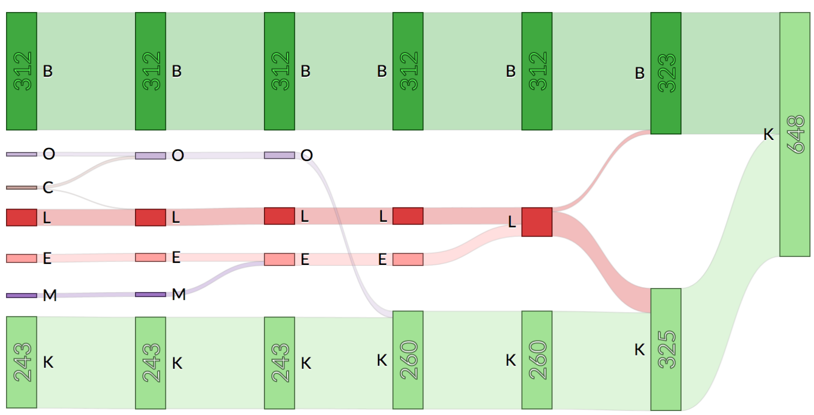


Figure 4: *Vote transfer Sankey diagram for a randomized exhaustible ballot election in which plan K wins.*

In 4, we see that in order for plan K to receive a majority of the ballots in the last round of the election, a significant portion of the previously exhaustible ballots must be awarded to K; even then, K's margin of victory is slim. Thus, we can conclude that the results of this election are not particularly susceptible to variation in preference order on exhaustible ballots.

3.2 Individuals

3.2.1 Simulated Elections with Exhaustible Ballots

For individual-choice elections, the results are more variable. For running simulated elections with individual MP preference ballots, we use the ratio weighting method and define majority support to mean $> 50\%$ of a party's MPs voting "Aye" (B.1.4).

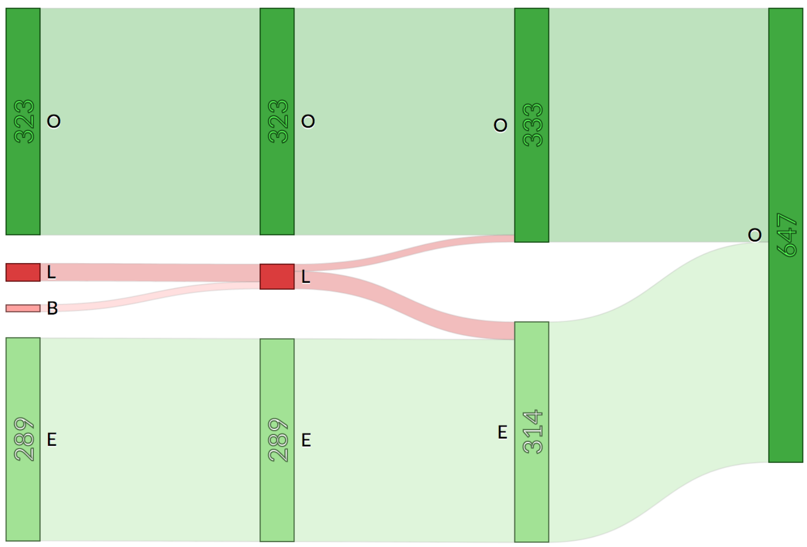


Figure 5: *Vote transfer Sankey diagram for the ratio-weighted individual MP election.*

The result here is resounding: there are only three elimination rounds in this election, and plan O never needs to receive additional ballots in order to win. O's measurable support among ballots can be attributed to its parent party, the Democratic Unionists.

Vote Percentage	Number of Occurrences	Percentage of Occurrences
100%	81	49%
90%	44	26%
88%	1	$< 1\%$
80%	25	15%
70%	4	2%
60%	12	7%

As mentioned in B.1.4, the ratio weighting method prefers parties who vote as blocs. More than 90% of the DUP's positive votes had more than or equal to eight of ten MPs vote "Aye," while 75% of the DUP's positive votes had more than or equal to nine of ten MPs vote "Aye." In fact, as is evidenced by the above table, the DUP votes as a consensus bloc on nearly half of all positive votes. Because of this, the DUP has an outsized influence over the results, which leads to plan O winning many of these races.

3.2.2 Simulated Elections with Randomized Ballots

Much like our experiment with party-line votes, we randomize exhaustible ballots in order to vary our results. The set of outcomes, however, is not as similar; as expected, when individual preference is accounted for, there is more volatility in the results.

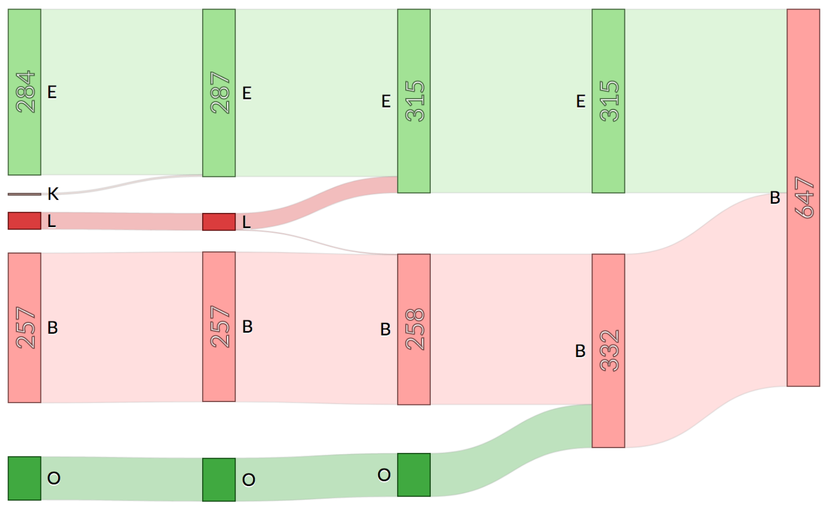


Figure 6: *Vote transfer Sankey diagram for the randomized ratio-weighted individual MP election. In this case, we show one of the elections in which plan B beats plan E.*

We find that in this randomized, ratio-weighted election, plan O only wins in 80% of the simulated elections, while plan B wins the remaining 20%.

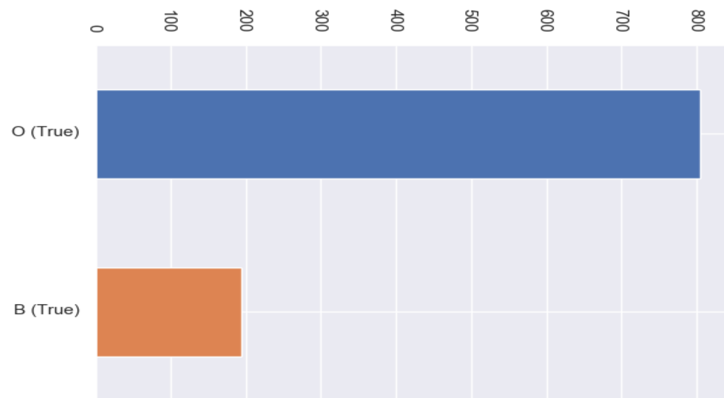


Figure 7: *Proportion of 1000 simulated elections won by each plan. The truth value next to each plan name indicates whether it was a Droop quota winner.*

This simulated election is much more variable than party-line votes. While most MPs tend to vote as their parties do, we see that "popular" plans according to parties are often not so among individual MPs. However, by varying some additional parameters, the results can shift wildly; these shifts and their results are detailed in C.1.1.

4 Conclusions

From the results in 3.1.1, it is clear that the largest party – in this case, the Conservatives – have the greatest influence over the results. Thus, if a Brexit plan were to be chosen via a ranked-choice vote with party-block, exhaustible ballots, then plan B – a No Deal Brexit – would win out. Even if we slightly modify this election by allowing the unfilled portions of exhaustible ballots to be randomized, there is still only a 2% chance that plan B does not win. Based on the former, we can conclude that if all MPs voted according to the preferences of their party at large, Brexit would come with No Deal.

However, if we take individual preference into account, we find that the results vary far more. In many elections, plan O – a "contingent preferential arrangements" plan introduced by the DUP – seems to win out; this can be attributed to the DUP's habit of voting as as bloc, so their influence in Parliament is strong. Other parties – notably the Conservative and Labour parties – have considerably less influence, largely due to their size. However, by toggling a few parameters, we can see (as evidenced by the tables in C.1.1) that the plan E – a Labour party plan – ends up winning quite a few elections as well. Overall, it seems that the tug-of-war between the DUP, the Conservative Party, and the Labour Party would likely end up catalyzing a plan E or plan O win.

A Brexit

A.1 Plans

This subsection details each of the Brexit plans.

A.1.1 Motion B – No Deal

This plan, originally proposed by Conservative MP John Baron, would allow the entirety of the United Kingdom to secede from the European Union without any agreements – immediately severing all ties. This plan was voted down, with 160 "Aye"s to 400 "No"s.

A.1.2 Motion C – Customs Union

Proposed by Conservative MP Ken Clarke, this plan forces the UK to negotiate a customs union with the EU, regardless of which plan is eventually adopted. Voted down 273 to 276.

A.1.3 Motion D – Common Market 2.0

Another Conservative plan, this time from Nick Boles, would have the UK join the European Free Trade Association (EFTA) and the European Economic Area (EEA). Furthermore, this would allow UK citizens to move and live freely throughout the EU. This plan was voted down 261 to 282.

A.1.4 Motion E – Confirmatory Public Vote

A joint venture from Labour party MPs Peter Kyle and Phil Wilson, this forces any Brexit plan adopted by Parliament to be put to a public vote. Voted down 280 to 292.

A.1.5 Motion G – Parliamentary Supremacy

Proposed by Joanna Cherry of the Scottish National Party, creates a contingency plan in the event that Parliament cannot reach a consensus. This process is:

1. if, by the time two days remain until the Brexit deadline, no deal has been reached, the government *must* file for a negotiation time extension;
2. if the EU refuses this extension request, Parliament would be required to choose between the No Deal plan *or* invoke Article 50 to halt Brexit;
3. if Article 50 is invoked, an acceptable UK-EU relationship is determined.

This plan was voted down 191 to 292.

A.1.6 Motion H – EFTA and EEA

George Eustice, a Conservative MP, proposed this Common Market 2.0-adjacent plan in which the UK rejoins the EFTA, but does not allow for a customs union between the UK and the EU. Voted down 64 to 377.

A.1.7 Motion K – Labour's Alternative Plan

Introduced by Labour party leader Jeremy Corbyn, this plan creates a permanent customs union, remains aligned with the EU single market, and provides additional consumer and environmental protections. Voted down 237 to 307.

A.1.8 Motion L – Revocation to Avoid No Deal

Also introduced by Joanna Cherry of the SNP, this plan proposes that, in the event no consensus plan can be reached, that a no-deal Brexit vote take place. Consequently, if the no-deal Brexit vote fails (i.e. Parliament opts to remain in the EU), Article 50 must be invoked. Voted down 184 to 293.

A.1.9 Motion O – Contingent Preferential Arrangements

A DUP plan introduced by Marcus Fysh, this plan stipulates that the UK must seek to find "preferential trading arrangements" in the event that no consensus plan is reached. Voted down 139 to 422.

B Preference Determination

B.1 Individuals

B.1.1 Weighting

Let $\mathcal{M} = \Omega$ be the set of motions, M the set of MPs, and P the set of parties. We define a weighting function w_* to be some function

$$w_* : M \times P \to \mathbb{R},$$

where $M \times P$ denotes the Cartesian product of M and P. In all probabilistic weighting methods, we take advantage of the fact that $\mathcal{M} = \Omega$ is a sample space by using probabilities to inform our weighting. Note that, in this context, the probability of some event E is a discrete probability such that

$$\mathbf{P}(E) = \frac{|E|}{|\Omega|},$$

with $\mathbf{P}(E) \in [0, 1]$.

B.1.2 Weighting: Intersection

This is the primary weighting function described in 2.2.1. More rigorously, this weighting function is defined (for a given $m \in M$ and $p \in P$) as

$$\begin{aligned} w_{\text{intersection}}(m, p) &= \mathbf{P}(m \cap p) \\ &= \frac{|m \cap p|}{|\Omega|} \\ &= \frac{\text{number of shared votes}}{\text{total number of votes}}. \end{aligned}$$

This weighting, however, is often a bad measure of party closeness. To illustrate, 8 depicts two dueling scenarios. On the left, we have an MP m_1 and a party p_1 that vote "Aye" infrequently, but nearly always vote "Aye" on the same motions.

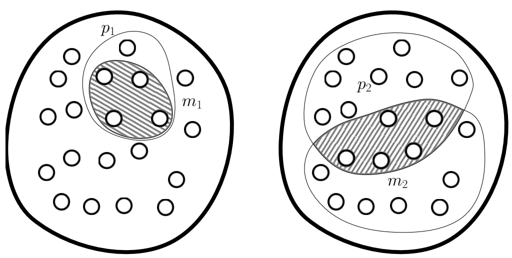


Figure 8

On the right, we have an MP m_2 and a party p_2 that vote "Aye" on quite a few motions, but vote together infrequently. Notice, however, that the intersection of m_2 and p_2 is actually *larger* than that of m_1 and p_1. In fact, if we use our earlier defined weight function $w_{\text{intersection}}$, we find that $w_{\text{intersection}}(m_1, p_1) = 1/5$ while $w_{\text{intersection}}(m_2, p_2) = 1/4$. In other words, $w_{\text{intersection}}$ punishes MPs and parties that vote together on a small number of motions, but can reward distant MPs and parties that vote on a large number of motions.

B.1.3 Weighting: Conditional

We define the conditional weighting function (for a given $m \in M$ and $p \in P$) to be

$$\begin{aligned} w_{\text{conditional}}(m, p) &= \mathbf{P}(m \mid p) \\ &= \frac{\mathbf{P}(m \cap p)}{\mathbf{P}(p)} \\ &= \frac{|m \cap p|}{|p|} \\ &= \frac{\text{number of shared votes}}{\text{number of } p \text{ votes}}. \end{aligned}$$

This attempts to accommodate for the shortcomings of the intersection-based weighting. Broadly, this weighting asks how likely it is for m to vote "Aye" on a motion given that p already supports it.

Using the example in B.1.2, we can see that $w_{\text{intersection}}(m_1, p_1) = 1/5$ and $w_{\text{intersection}}(m_2, p_2) = 1/4$.

However, if we use conditional weighting, we see that $w_{\text{conditional}}(m_1, p_1) = 4/5$, while $w_{\text{conditional}}(m_2, p_2) = 5/13$. While this is an improvement, a similar shortcoming can be pointed out, as is shown in 9. Here, we have a similar situation to 8: p_1 and m_1 remain the same, but m_3 and p_3 represent a new pairing of MP and party. In this example, we see that m_3 actually votes against p_3 far more often than they vote with p_3. Echoing the first scenario, we see that $w_{\text{intersection}}$ for m_1 and p_1 is the same as that for m_3 and p_3; thus, the intersection weight tells us that m_1 and p_1 are as likely to vote together as m_3 and p_3 are.

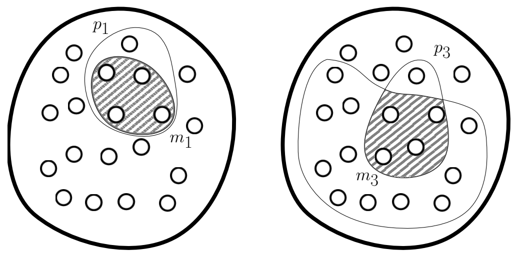


Figure 9

In this case, however, $w_{\text{conditional}}(m_1, p_1) = 4/5 = w_{\text{conditional}}(m_3, p_3)$; in other words, the conditional weighting tells us exactly the same thing that the intersection weighting does. Thus, we conclude that $w_{\text{conditional}}$ punishes MPs and parties that consistently vote together on a small number of motions, but can reward MPs and parties who vote disparately on most motions but together on a select few.

B.1.4 Weighting: Ratio

We define the ratio weighting function (for a given $m \in M$ and $p \in P$) to be

$$
\begin{aligned}
w_{\text{ratio}}(m, p) &= \frac{\mathbf{P}(m \cap p)}{\mathbf{P}(m \oplus p)} \\
&= \frac{|m \cap p|}{|(m \setminus p) \cup (p \setminus m)|} \\
&= \frac{|m \cap p|}{|(m \cup p) \setminus (m \cap p)|} \\
&= \frac{\text{number of shared votes}}{\text{number of not shared votes}}.
\end{aligned}
$$

This is a slight improvement on our conditional weighting (and directly addresses the problem with intersection weighting) by taking the ratio of shared to not shared votes. If an MP and a party are close, they should share a lot of votes and not share a small amount of votes. If an MP and a party are distant, they should have few shared votes and a large number of non-shared votes.

Consider our previous examples in figures 8 and 9. In each of these cases, the weights applied do not accurately reflect the voting tendencies of the MP/party pairs.

Table 2

MP, party	$w_{\text{ratio}}(m_i, p_i)$
m_1, p_1	$4/1 = 4$
m_2, p_2	$5/15 = 1/3$
m_3, p_3	$4/12 = 1/3$

The ratio weight, as defined above, is designed to reward MP/party pairs with a large number of shared votes and a small number of non-shared votes, while punishing MP/party pairs who share few votes and vote against each other often. This is reflected in table 2. Furthermore, this method of weighting benefits parties who often vote as blocs; in other words, a party who votes on many motions with nearly all party members voting "Aye" will receive greater weight than parties who often split votes.

B.1.5 Weighting: Party Size

In addition to weighting by some measure of shared/not shared votes, we can also give large parties more (or less) influence by way of increasing (or decreasing) their weight based on the size of the party. Suppose some party p has n members. Then, we can introduce the following size-based weights:

$$s_* : \mathbb{N} \to \mathbb{R},$$

$$s_*(n) = \begin{cases} s_{\text{log10}}(n) & = \log_{10} n \\ s_{\text{log2}}(n) & = \log_2 n \\ s_{\text{ln}}(n) & = \ln n \\ s_{\text{raw}}(n) & = n \end{cases}$$

We can also show how these weights scale with the size of our input. Suppose that $n = 10$ (i.e. p has 10 members).

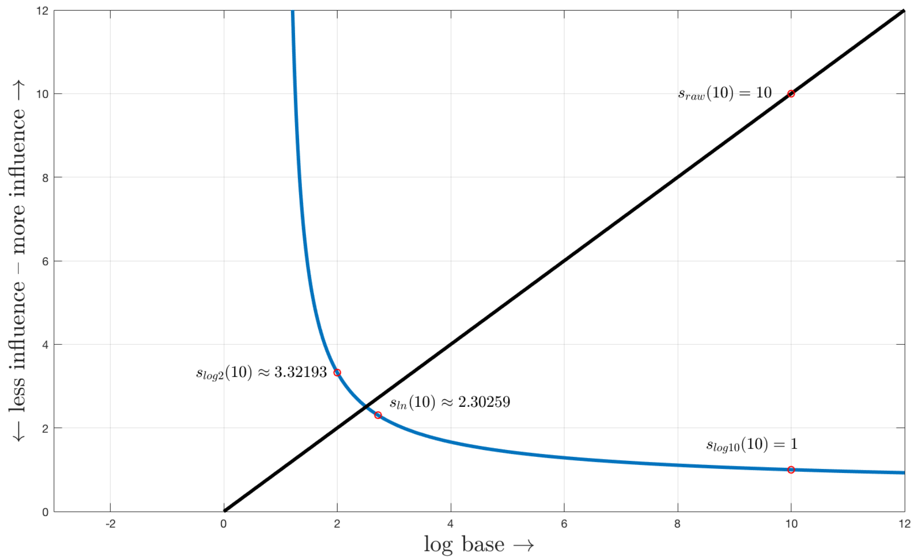


Figure 10: *Depiction of the functions* $f(x) = \log_x(10)$ *in blue and* $s_{raw}(x) = x$ *in black, with markers denoting the values of our selected weight functions.*

Figure 10 shows log bases plotted against the function $f(x) = \log_x(10)$, with markers denoting the values of our individual weight functions. (For completeness, we also include the function $s_{\text{raw}}(x) = x$, in black.) From this, we see that larger parties (i.e. parties with large n) are awarded an increasingly smaller influence weight as the log base increases. However, for s_{raw}, the influence weight is simply equal to the size of the party.

Further, we may use these weights to scale our probabilistic ones. In this way, we can associate some total weight W_{total} to an MP m such that

$$W_{\text{total}} = s_a(n) \cdot w_b(m, p),$$

where a and b denote a choice of size- and vote share-based weighting method.

B.1.6 Plan Choice Algorithm

The plan choice algorithm, as outlined in 2.2.2, is reasonably simple. Due to restrictions on the way the data are formatted, however, we cannot simply vectorize weights and take dot products; in fact, we are forced to use an iterative method. However, (ignoring time complexity of floating-point multiplication), the iterative method is linear in the number of MPs. Below is the pseudocode for implementing this algorithm iteratively. Define $PREFS_m$ and $SUPPORT_p$ to be hashmaps such that

$$m \in M,\ PREFS_m : P \to \mathbb{R},$$

$$p \in P,\ SUPPORT_p : \mathcal{B} \to \mathbb{R},$$

with $\mathcal{B}$ the set of Brexit plans. Note that $PREFS_m$ is simply a mapping from parties to their weights (for MP m), and $SUPPORT_p$ is a mapping from parties to their support scores for plans. Now, we can describe the algorithm with the following:

Data: $PREFS_*$, $SUPPORT_*$, $\mathcal{B}$
Result: Set of ordered plan rankings for each MP.

```
RANKINGS := [ ];
for each m in M do
    /* plan_weights maps plans to their current weight value.
       Each plan starts at a weight of 0, but these weights will be
       progressively added to.                                      */
    plan_weights := {b → 0 ∀b ∈ B};

    for each p in P do
        weight_m := PREFS_m[p];
        for each b in B do
            support_p,b := SUPPORT_p[b];
            density := (weight_m · support_p,b);
            plan_weights[b] := plan_weights[b] + density;
        end
    end
    preferences := sort(plan_weights);
    RANKINGS = RANKINGS + preferences;
end
return RANKINGS;
```

Also note that $sort(\circ)$ sorts the $plan\_weights$ mapping's keys by their value; i.e. the key with the greatest value will be at the beginning of the returned collection, and so on. This gives us a ranked plan list, as the keys of $plan\_weights$ are the names of the plans themselves.

B.1.7 Plan Choice Algorithm: Alternate Method

As a precursor to B.1.6, an alternate plan choice algorithm which selects plans in a pseudo-RCV manner was developed. This method is deprecated in favor of that detailed in B.1.6, but played an important part in the development of this project. Essentially, this algorithm relies on choosing the most popular plan at a given position. Consider the election laid out in 2.2.2.

Party	Weight	Rank 1	2	3	4
Interplanetary	0.94	P_3	P_1	P_2	P_4
Hooky	0.86	P_1	P_2	P_3	P_4
Anti-Leadership	0.307	P_4	P_3	P_2	P_1
m	–	P_1	P_2	P_3	P_4

The algorithm progresses by column, checking which plan has the most collective weight; for the "Rank 1" column this is P_3, for the "2" column this is P_1, for the "3" column this is P_2 (as it has a combined weight of $0.94 + 0.307 = 1.247$), and for the "4" column this is P_4 (as it has a combined weight of $0.94 + 0.86 = 1.8$). This algorithm is more cumbersome and relies on the absolute positions of plans in each party's plan ranking – it does not take the support score of a party into account.

B.1.8 Random Weighting

Here, we describe the procedure for providing random party weights for individual MPs. It is reasonably simple: given some MP m and a set of party weights W_m for m, we find the weights $w \in W$ such that $w = 0$; call this subset $W_0 \subseteq W$. Then, we find $w_{\text{min}} = \text{min}\{W \setminus W_0\}$. Finally, for each $w \in W_0$, we assign a uniformly random weight $w_{\text{random}} \in [0, w_{\text{min}})$.

C Results

C.1 Individuals

C.1.1 Nonrandom Results

The below tables show the winners of individual nonrandom ranked-choice elections where the weighting methods used correspond to the rows and columns; for example,

	–	weight A
weight B	winning Brexit plan w/ B alone	winning Brexit plan w/ A and B

	–	s_{log10}	s_{ln}	s_{log2}	s_{raw}
$w_{intersection}$	E	O	O	O	B
$w_{conditional}$	E	O	O	O	B
w_{ratio}	O	O	O	O	B

Table 3: *Tabulated results of an individual election where "majority support" is defined to be 30%, 60%, or 66%.*

	$-$	s_{log10}	s_{ln}	s_{log2}	s_{raw}
$w_{intersection}$	O	O	O	O	B
$w_{conditional}$	O	O	O	O	B
w_{ratio}	O	O	O	O	B

Table 4: *Tabulated results of an individual election where "majority support" is defined to be 50% or 75%.*

	$-$	s_{log10}	s_{ln}	s_{log2}	s_{raw}
$w_{intersection}$	E	E	E	E	B
$w_{conditional}$	E	E	E	E	B
w_{ratio}	O	O	O	O	B

Table 5: *Tabulated results of an individual election where "majority support" is defined to be 50% or 66% and we use only divisions numbered 471-689.*

	$-$	s_{log10}	s_{ln}	s_{log2}	s_{raw}
$w_{intersection}$	E	E	E	E	B
$w_{conditional}$	E	E	E	E	B
w_{ratio}	E	E	E	E	B

Table 6: *Tabulated results of an individual election where "majority support" is defined to be 50% and we count "No Vote Recorded" as well as "Aye" votes.*

	$-$	s_{log10}	s_{ln}	s_{log2}	s_{raw}
$w_{intersection}$	E	E	E	E	E
$w_{conditional}$	E	E	E	E	E
w_{ratio}	E	E	E	E	B

Table 7: *Tabulated results of an individual election where "majority support" is defined to be 66% and we count "No Vote Recorded" as well as "Aye" votes.*