

Biostatistical Methods I (BIOS 5710)
Breheny

Assignment 7

Due: Wednesday, October 29

1. In lab, we saw that the confidence interval derived from the pivotal quantity $(n-1)S^2/\sigma^2 \sim \chi_{n-1}^2$ did not hold up well for non-normally distributed data with $n = 25$. Does the accuracy of this interval improve (i.e., get closer to the nominal level) as n gets larger? Try this out with a simulation and then explain why the accuracy of the interval is improving, or why it isn't. For this simulation study, set `replace=TRUE` when using `sample`; this doesn't make much difference when n is small, but if you increase n into the thousands, it makes a big difference. If `replace=FALSE` and n is large, the sample is artificially close to the population (because our population here is fairly small) and coverage is artificially inflated.
2. The course website contains a data set that lists the sex and ages at death (in days) of 16 children who died of sudden infant death syndrome in Seattle from 1973-1982.
 - (a) Create a table that lists the mean and standard deviation for age at death for males and females, along with the sample size for each group. Based on the table, would you expect Student's t -test and Welch's t -test to give similar answers?
 - (b) Test whether or not the difference in age at death for males and females could be due to chance using Student's t -test.
 - (c) Test whether or not the difference in age at death for males and females could be due to chance using Welch's t -test.
 - (d) Construct a 95% confidence interval for the difference in age at death for males and females, based on the assumption of equal variability in males and females.
 - (e) Construct a 95% confidence interval for the difference in age at death for males and females, without assuming that variability is equal in males and females.
 - (f) Which approach (Welch/Student, equal/unequal SD) do you think is most appropriate in this circumstance? Why?
3. In class, we said that greatest reduction in standard error happens when the sample sizes of the two groups are balanced. This is true under the equal variance assumption, but what about when the variances of the two groups are different? Suppose group 1 has variance σ^2 and group 2 has variance $r\sigma^2$ (i.e., its variance is r times larger than group 1) and that the total sample size is fixed. Is $n_1 = n_2$ still the most efficient design? If not, what is the proper ratio between n_1 and n_2 ?

Hint: The standard method for solving an optimization problem subject to a constraint (i.e., minimize standard error subject to the constant $n_1 + n_2 = N$) is using the method of Lagrange multipliers. I'm sure you have seen Lagrange multipliers in your past – if you've forgotten how this method works, consult any Calculus textbook.
4. Show that if observations from group 1 follow a $N(\mu_1, \sigma^2)$ distribution and observations from group 2 follow a $N(\mu_2, \sigma^2)$ distribution, then

$$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$$

follows a t distribution with $n_1 + n_2 - 2$ degrees of freedom. You may use any result that is stated in the class notes (except, obviously, the one that states this result).

5. In class, we discussed the determination of sample size from the perspective of hypothesis testing (power). It is also possible to decide sample size on the basis of confidence interval considerations. Consider the cholesterol example we discussed in class where $\sigma = 36$ and we are planning a two-sample study of treatment vs. control where each group will have the same sample size n . For the sake of this problem, treat σ as a known constant.
 - (a) Suppose we carry out the study with a sample size of $n = 50$ per group. What will the width of a 95% confidence interval be?
 - (b) How large must the sample size be in order to ensure that the width of the confidence interval will be smaller than 10 mg/dL?
 - (c) If we wanted to take into account the fact that the standard deviation is an unknowable parameter and must be estimated, what two things must change in your approach to (a) and (b)?
6. Suppose we are planning a crossover study of a new therapy and we expect that 70% of patients will do better on the new therapy than the control. For (a)-(c) below, base your answers on the exact distribution of patients who will do better on one therapy than the other.
 - (a) Suppose $n = 25$. What are the critical values for testing $\pi = 0.5$ at the $\alpha = 0.05$ level?
 - (b) For $n = 25$, what is the power of our study to reject $\pi = 0.5$ at the $\alpha = 0.05$ level?
 - (c) How large must n be in order to achieve 90% power?