BIOS:4120 - Lab 6 Solution

February 20-21, 2024

Problem 1

(a) Are years of study mutually exlusive?

Yes, the years of study are mutually exclusive. The players cannot belong to more than one of the categories. (Example: One cannot be both a freshman and senior).

(b) Probability player is not a senior

Note: There are multiple ways to get the solution.
Let S be the probability that the football player is a senior.

$$P(S^c) = \frac{99 - 21}{99} = 0.7878$$
$$P(S^c) = 1 - P(S) = 1 - \frac{21}{99} = 0.7878$$

(c) Probability a player is a sophomore, given they are not a senior

Let $Soph$ be the probability that the football player is a sophomore.
Let S be the probability that the football player is a senior.

$$P(Soph|S^c) = \frac{12}{99 - 21} = 0.15$$
$$P(Soph|S^c) = \frac{P(Soph \cap S^c)}{P(S^C)} = \frac{P(Soph)}{P(S^c)} = \frac{0.12}{0.78} = 0.15$$

(d) Probability of selecting two juniors

With replacement:

$$\frac{34}{99} \times \frac{34}{99} = 0.1179$$

Without replacement:

$$\frac{34}{99} \times \frac{33}{98} = 0.1156$$

Problem 2

The table is filled in using the given information as follows:

	Periodontal Status			
Gender	**Healthy**	**Gingivitis**	**Perio**	**Total**
Male	1147	926	936	3009
Female	2603	1495	920	5018
Total	3750	2421	1856	8027

Let's denote the following events:

- M (Person is male)
- F (Person is female)
- H (Person is healthy)
- G (Person has gingivitis)
- Per (Person has periodontal disease)

(a) Probability that the participant is male or healthy

$$P(M \cup H) = \frac{3009 + 3750 - 1147}{8027} = 0.699$$

(b) Probability of being female and having either gingivitis or periodontal disease

Using the table to solve this problem, we can find the number of people that are female and have gingivitis and add that to the number of people who are female and have periodontal disease, and then divide that by the total number of people:

$$\frac{1495 + 920}{8027} = 0.301$$

(c) Probability of an individual having periodontal disease given that she is female

Using the table to solve this problem, we want to look at the "given" information. So we want to look at the female row. From this row, we want to find the number of females that have periodontal disease (920) and divide that by the row total (5018).

$$\frac{920}{5018} = 0.183$$

We can also use the formula for conditional probability if we want to do the problem more formally.

$$P(Per|F) = \frac{P(Per \cap F)}{P(F)} = \frac{\frac{920}{8027}}{\frac{5018}{8027}} = 0.183$$

Problem 3

In this problem, let H denote the event of having HIV, and T the event of testing positive.

(a) Express given information in terms of H and T

Given information:

$$P(T|H) = 0.95$$
$$P(H) = 0.001$$

(b) Probability of HIV and positive test

The probability of someone having HIV and testing positive is given by the product of the probabilities of each event:

$$P(H \cap T) = P(T|H) \times P(H) = 0.95 \times 0.001 = 0.00095$$

To interpret this as a rate per 10,000 people, we multiply by 10,000:

$$0.00095 \times 10,000 = 9.5 \text{ cases per 10,000 people}$$

(c) Probability of negative test, given person has HIV

The probability of testing negative given that someone has HIV is the complement of testing positive given they have HIV:

$$P(T^c|H) = 1 - P(T|H) = 1 - 0.95 = 0.05$$

Problem 4

Let's denote the following events:

- $D+$ (Person has breast cancer.)
- $D-$ (Person does not have breast cancer.)
- $T+$ (Person tests positive for breast cancer.)
- $T-$ (Person tests negative for breast cancer.)

(a)

Given information:

$$\begin{aligned} P(D+) &= 0.01 \quad \text{(prevalence of breast cancer)} \\ P(D-) &= 0.99 \quad \text{(prevalence of no breast cancer)} \\ P(T+|D+) &= 0.90 \quad \text{(sensitivity)} \\ P(T-|D+) &= 0.10 \quad \text{false negative, complement to sensitivity} \\ P(T+|D-) &= 0.09 \quad \text{(false positive rate)} \\ P(T-|D-) &= 0.91 \quad \text{(specificity, complement to false positive rate)} \end{aligned}$$

(a) Setting up Bayes' Rule

Bayes' Rule states:

$$P(D+|T+) = \frac{P(T+|D+) \cdot P(D+)}{P(T+|D+) \cdot P(D+) + P(T+|D-) \cdot P(D-)}$$

(b) Plug in and Calculate

$$P(D+|T+) = \frac{0.90 \times 0.01}{(0.90 \times 0.01) + (0.09 \times 0.99)} = \frac{0.009}{0.0981} \approx 0.0917$$

(c) Interpretation

The probability that a person has breast cancer given that they tested positive is approximately 0.0917, or about 9.17%. This indicates that even if someone tests positive for breast cancer, there is still a relatively low chance that they actually have the disease.

Problem 5

Let's denote the following events:

- H: Event it rains in Hawaii
- I: Event it rains in Iowa City

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\begin{aligned}
P(H \cap I) &= P(H) \times P(I) = 0.6 \times 0.2 = 0.12 \\
P(H \cap I^c) &= 0.6 - 0.12 = 0.48 \\
P(I \cap H^c) &= 0.2 - 0.12 = 0.08 \\
P(H \cup I)^c &= P(H^c \cap I^c) = 1 - 0.48 - 0.08 - 0.12 = 0.32
\end{aligned}
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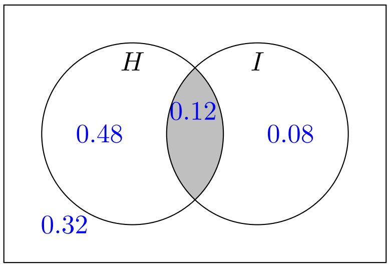

Problem 6

Let's denote the following events:

- F: Drawing a face card (jack, queen, or king).
- R: Drawing a red card.
- 7: Drawing a 7.

(a) Probability of Drawing a Face Card

The probability of drawing a face card is:

$$P(F) = \frac{12}{52} = 0.2307$$

(b) Probability of Drawing a Red Face Card

The probability of drawing a red face card is the intersection of drawing a face card and drawing a red card:

$$P(F \cap R) = P(F) \times P(R) = \frac{12}{52} \times \frac{26}{52} = \frac{3}{13} \times \frac{1}{2} = \frac{3}{26} = 0.1153$$

Alternatively, think about how many cards in the deck would be both red and face cards. There are 3 face cards per suit and 4 suits total. However, only two of the suits are red (hearts and diamonds), so there would be 6 total face cards that are red out of the 52 total cards:

$$P(F \cap R) = \frac{6}{52} = 0.1153$$

(c) Probability of Drawing a Red Card or a Face Card

The probability of drawing a red card or a face card is the sum of their individual probabilities minus the probability of drawing both:

$$P(R \cup F) = P(R) + P(F) - P(R \cap F)$$

$$P(R \cup F) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = 0.6154$$

(d) Probability of Drawing a Red Card, a Face Card, or a 7

Similarly, the probability of drawing a red card, a face card, or a 7 is:

$$P(R \cup F \cup 7) = P(R \cup F) + P(7) - P(R \cap 7)$$

$$P(R \cup F \cup 7) = \frac{32}{52} + \frac{4}{52} - \frac{2}{52} = \frac{34}{52} = 0.6538$$

In the previous question, we found the probability of red card or face card, we just need to add in the probability of it also being a 7, but when we add that in, we need to subtract the probability that it is a red 7 or a face card that is 7. The probability that it is a face card and 7 is 0, so that's why it was not written above.

If we wanted to write out the full formula and expand the addition rule, it would look like this:

$$P(R \cup F \cup 7) = P(R) + P(F) + P(7) - P(R \cap F) - P(R \cap 7) - P(F \cap 7) + P(R \cap F \cap 7)$$