Testing for Spatial-Autoregressive Lag versus (Unobserved) Spatially Correlated Error-Components

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ABSTRACT: One of the central challenges to empirical inference in the context of potentially interdependent observations, known as *Galton's Problem*, is the difficulty distinguishing spatial correlation in outcomes due to the observed units' exposure to spatially correlated shocks ('common exposure') from spatially dependent outcomes due to interdependence ('spillovers' or 'contagion') among units. The applied researcher's first, and until recently only, defense against confusing these substantively importantly different processes empirically has been to control as best possible with observable regressors and/or groupwise dummy-variables for correlated-shocks processes when estimating interdependence models (spatial autoregression). Specifying empirical models and measures as precisely and powerfully as possible remains the optimal practice, but these strategies cannot guard fully against the possibility of exposure to *unobserved* exogenous shocks that are distributed spatially in some manner not fully common to some set of units (which fixed or random group effects could account) or fully controlled by observable exogenous factors (control variables), but are instead distributed across units more similarly to the pattern by which the outcome is contagious. This paper reviews recent developments in testing for three data-generating processes (models) that span those possibilities: random effects by spatial units ('groupwise [random or fixed] effects'), time-invariant group effects with interdependence across units ('spatially interdependent group effects'), and time-varying spatial effects (i.e., the spatial-error or spatial-lag models.) The paper explores by Monte Carlo analyses how well Anselin et al. (1996)'s robust Lagrange-multiplier tests of spatial-autoregressive lag versus spatial-autoregressive error processes may distinguish these three models as well, and how effectively spatial-group dummy-variables or Griffith's (2000) eigenvector spatial-filtering may control for them as alternatives in spatial-autoregressive models of interdependence. These analyses show the robust LM tests can be constructive and informative also in distinguishing these alternative sources of spatial association, and that eigenvector spatial-filtering offers an effective control in some conditions, and does so more generally than do spatial-group dummy-variables. (In fact, eigenvector spatial-filtering would seem to be a generalization that subsumes spatial dummy-variables, something not previously noticed in the literature.) The paper concludes with proposing an overall approach to empirical analysis of interdependence that includes as an important step using these tests and controls to provide direct answer to the question posed by *Galton's Problem*: *common exposure or contagion?*

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I. Introduction: Spatial Interdependence and Spatial Association

Spatial/Spatiotemporal interdependence—i.e., that the outcomes, actions, or choices of some unit-times depend on those of others—is substantively and theoretically ubiquitous and often centrally important in outcomes of interest across the social sciences. Likewise, empirically, spatial association—correlation of outcomes across units by some pattern—is ubiquitous and often quite strong. However, outcomes may exhibit spatial association for at least two distinct reasons.[1] Units may be responding similarly to similar exposure to similar

[1] A third possibility, which we do not address here (but see Franzese et al. 2010, 2012) arises when the outcome or explanatory variables affect the variable along which clustering occurs (*selection*: e.g., homophily). For instance, in a classic sociological example, we may observe clusters of smoking and of non-smoking friends because friends acquire the behavior from each other (*contagion*), because smokers and non-smokers experience different sets of stimuli (similar sociodemographics, e.g.) (*common exposure*), or because (non)smokers are more likely to become

exogenous internal/domestic or external/foreign shocks or stimuli (*common exposure*); second, units' outcomes may depend on other units' outcomes (*contagion*). Severe empirical difficulties confront the accurate estimation and distinction of these importantly distinct alternative sources of spatial association.

In previous work (Franzese & Hays 2004, 2006, 2007, 2008ab), we derived analytically, for spatial or spatiotemporal linear-regression simple cases, the biases given interdependence of non-spatial least-squares (LS) and of spatial least-squares (S-LS: i.e., of omitting spatial lags or of including them but ignoring their endogeneity), and explored in simulations under richer, more-realistic, and limited-sample conditions the properties of these biased LS and S-LS estimators and of the consistent and asymptotically efficient spatial method-of-moments (S-MoM: S-IV, S-2SLS, S-GMM) and maximum-likelihood (S-ML) estimators. Our central findings in these analyses were that LS, by ignoring spatial interdependence, suffers omitted-variable biases that foster overestimation of non-spatial effects, i.e., unit-level (domestic, individual) and contextual (exogenous-external) effects. These biases quickly grow substantively sizeable at even very modest interdependence-strength ($\rho>.1\pm$) and become enormous at greater ρ, and the biases generally accrue across the non-spatial regressors proportionately to how closely these regressors' spatial distribution across units parallel the connectivity pattern of the interdependence. Standard errors for these overestimated effects are also dramatically underestimated in these ranges, and PCSE (Beck and Katz 1995, 1996) offers little amelioration. Given any noticeable interdependence, then, non-spatial LS is an unmitigated disaster. In converse, inadequate modeling or control for non-interdependence aspects of the data-generating process will induce over-estimation of spatial-interdependence strength, ρ, especially insofar as

and/or remain friends with (non)smokers (*selection*). This paper addresses distinguishing the first two of these, which is *Galton's Problem*.

these omitted or inadequately modeled or controlled factors exhibit distribution across units that relates in some way to the connectivity matrix by which interdependence among units (contagion) manifests.[2]

In sum, relative omission or misspecification of the spatial or non-spatial component of the model fosters underestimation of the strength of the relatively poorly specified component and overestimation of the better-specified component. Substantively for political scientists, then, relatively poor specifications of domestic (or micro/individual-level) components or of exogenous-external (macro/contextual-level) components (common shocks/stimuli) will tend to bias conclusions to favor contagion, and *vice versa*. The most important issue, then, is adequate modeling or control both of interdependence, including accurate and empirically powerful specification of **W**, and of the non-spatial component of the model (i.e., unit-level and exogenous-external factors), including, centrally importantly to this paper, potentially unobserved such factors.[3] Quite simply: unobserved heterogeneity will bias estimates of the strength interdependence insofar as these unobserved (or insufficiently modeled or controlled) effects distribute across units in a pattern related to that of the connectivity underlying the interdependence. *Common exposure* in unobservables is one important threat (a *Galton's Problem*) to valid empirical inference regarding spatial interdependence.

Furthermore, there are crucially important substantive distinctions between *interdependence*, i.e. contagion and spillovers, and *common exposure*, and in whether contagion

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[2] S-LS, conversely, suffers simultaneity biases that foster mis-estimation, usually over-estimation, of contagion strength, usually inducing oppositely signed errors for (i.e., underestimation of) non-spatial factors' roles. These simultaneity biases generally remain mild at weaker interdependence ($\rho<.25\pm$), and S-LS is also rather efficient, but standard-error accuracy is very poor in smaller-T samples (as, in the extreme, in pure cross-sections). Similarly in logic, conversely in direction, to the omitted-variable biases of LS, the simultaneity bias that typically inflates estimated interdependence in S-LS induces corresponding attenuation biases in the estimates of non-spatial explanatory roles, especially for factors exhibiting spatial correlation most similar to the pattern of dependent-variable interdependence.

[3] Selecting appropriately consistent estimators (and which consistent estimator) also become(s) important as interdependence strengthens.

occurs in the entire outcome variable or only in the stochastic or error term.

Interdependence, i.e., contagion, in the outcome variable implies what is called the *spatial-autoregressive lag* (SAR) model in spatial-econometrics; in a cross-section, that model is:

$$\text{SAR:} \quad \underset{N\times 1}{\mathbf{y}} = \rho \underset{N\times N}{\mathbf{W}} \underset{N\times 1}{\mathbf{y}} + \underset{N\times k}{\mathbf{X}} \underset{k\times 1}{\boldsymbol{\beta}} + \underset{N\times 1}{\boldsymbol{\varepsilon}} \tag{1}$$

Substantively, notice that the spatial autoregression implied by interdependence implies feedback multiplier effects, such that a change in any exogenous factor(s) x, in any spatial unit(s), $\{i\}$, affects all the units, $\{j\}$, to which those initially affected are connected according to $\mathbf{W}$, which affects all the units to which those $\{j\}$ are connected, including reflecting back onto the originally affected $\{i\}$, and so on. These spatial responses can be seen by differentiating the reduced form of the SAR model (1):

$$\begin{aligned}
&\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \\
&\Rightarrow\ \mathbf{y} - \rho\mathbf{W}\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}\ \Rightarrow\ (\mathbf{I} - \rho\mathbf{W})\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \\
&\Rightarrow\ \mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}) \qquad \text{(reduced form)} \\
&\Rightarrow\ d\mathbf{y} = (\mathbf{I} - \rho\mathbf{W})^{-1}(d\mathbf{X})\boldsymbol{\beta} \qquad \text{(spatial response)}
\end{aligned} \tag{2}$$

These are models in which, to reverse a popular advertisement for Las Vegas tourism, "What happens in Vegas doesn't stay in Vegas." Indeed, *none* of what happens in Vegas stays in Vegas; it all spreads everywhere (connected by the expansion: $(\mathbf{I} - \rho\mathbf{W})^{-1} = \mathbf{I} + \rho\mathbf{W} + \rho^2\mathbf{W}^2 + \rho^3\mathbf{W}^3 + \ldots$), including feeding back into Vegas (amplifying the accumulated response (for positive feedback) relative to the initial $d\mathbf{X}$, rather like in certain bachelor-party movies set in Vegas).

Common exposure to spatially associated exogenous factors, observed or unobserved, on the other hand, generally does not have quite the same extent or breadth of feedback and multiplier implications. To illustrate, consider a model without spatial interdependence in the outcome, but with spatial association among some of the exogenous explanators, $\mathbf{X}_2$, induced by spillovers of those factors $\mathbf{X}_2$ from units $\{i\}$ into proximate units $\{j\}$ according to connectivity matrix $\mathbf{W}$:

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{W}\mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon} \qquad (3).$$

In this case, some of what happens in Vegas spills over into adjacent (according to $\mathbf{W}$) areas, and that's the end of the movie:

$$d\mathbf{y} = \mathbf{W}d(\mathbf{X}_2)\boldsymbol{\beta}_2 \qquad (4),$$

and if these factors subject to spillovers are unobserved, then $\mathbf{W}\mathbf{X}_2\boldsymbol{\beta}_2$ becomes part of the error term and the bias from omitting this unobserved spatial-heterogeneity is given by the standard omitted-variable-bias formula:

$$\begin{aligned} &bias(\boldsymbol{\beta}_1) = \boldsymbol{\beta}_2\mathbf{F}_{\mathbf{X}_1}^{\mathbf{WX}_2}, \text{ where} \\ &\quad \mathbf{F}_{\mathbf{X}_1}^{\mathbf{WX}_2} \text{ is the regression of each column of } \mathbf{WX}_2 \text{ on all columns of } \mathbf{X}_1 \end{aligned} \qquad (5).$$

Alternatively, the spatial association in $\mathbf{X}_2$, observed or unobserved, may arise from a contagion process, with its implied autoregressive form:

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon} \text{ , where } \mathbf{X}_2 : \mathbf{X}_2^* = \rho\mathbf{W}\mathbf{X}_2^* + \nu, \ \Rightarrow \mathbf{X}_2 = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{X}_2^* \qquad (6).$$

In this case, the units' responses to some $d\mathbf{X}_2^*$ would be their response to the post-feedback and multiplier transformed shock, $(\mathbf{I} - \rho\mathbf{W})^{-1}d\mathbf{X}_2^*$, but only this component induces responses exhibiting spatial feedback and multiplier transformations in this way. If these spatial-autoregressive factors, $\mathbf{X}_2$, are unobserved, then the model representing the process is the classical spatial-autoregressive-error model (SEM) from spatial econometrics:

$$\text{SEM: } \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \text{ with } \boldsymbol{\varepsilon} = \lambda\mathbf{W}\boldsymbol{\varepsilon} + \mathbf{u} \quad \Rightarrow \mathbf{y} = \mathbf{X}\boldsymbol{\beta} + (\mathbf{I} - \lambda\mathbf{W})^{-1}\mathbf{u} \qquad (7).$$

Analogously, the spillover spatial-association in unobservables model (3) above is equivalent to the spatial-autoregressive moving-average (SARMA) model: $\mathbf{y} = \rho\mathbf{W}_1\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, with $\boldsymbol{\varepsilon} = \lambda\mathbf{W}_2\mathbf{u} + \mathbf{u}$. In either case, if these unobserved spatially associated factors covary with other regressors, bias will arise according to the omitted-variable-bias formula (5), and inefficiency and incorrect standard errors as well, and if the unobserved spatially associated factors do not covary with other regressors, then estimates ignoring them will incur inefficiency alone.

Distinguishing spatial-autoregressive lag processes (i.e., contagion) from spatial association in outcomes arising from exposure to spatially correlated exogenous factors, is therefore crucially important not only to accurate estimation but also to correct substantive interpretation of those estimates in terms of what they imply for responses to counterfactuals of interest (i.e., $d\mathbf{y}/d\mathbf{X}$).

In summary, one great challenge to empirical inference and to substantive interpretation in contexts of potentially interdependent observations is the difficulty distinguishing spatial association in outcomes arising from *common exposure* to spatially correlated shocks from spatially dependent outcomes due to *contagion*, with its implied feedback and multiplier effects. Until recently, the applied researcher's only defense against confusing these substantively importantly different processes empirically has been to model the common exposure as best possible with observable regressors and/or to control by groupwise dummy-variables for (fully) common shocks when estimating the spatial-autoregressive models that properly reflect interdependence. Specifying empirical models and measures as precisely and powerfully as possible remains, as always, optimal practice, but these strategies cannot guard fully against the possibility of exposure to unobserved exogenous shocks that are distributed spatially in some manner not fully common to some sets of units (which fixed or random group effects could account) or fully controlled by observable exogenous factors, but are instead distributed across units in some other manner similarly to the pattern by which the outcome is contagious.

The next section reviews recent developments in controlling and/or testing for three data-generating processes (DGPs) that span those possibilities: random effects by spatial units ('groupwise [random or fixed] effects'), time-invariant group effects with interdependence across units ('spatially interdependent group effects'), and time-varying spatial effects (i.e., the spatial-error or spatial-lag models.) The third section explains Anselin et al. (1996)'s robust Lagrange-multiplier tests of spatial-autoregressive lag versus spatial-autoregressive error, each of which tests its form of spatial dependence against independence but without power against the other form of dependence, and Griffith's (2000, 2002, 2003) eigenvector spatial-filtering process, which derives from **W** an optimal set of linear controls for additively separable spatial association. Section IV conducts Monte Carlo analyses to explore how well these robust Lagrange-multiplier tests of SAR and/vs. SEM may distinguish these three DGPs as well, and how effectively spatial-group dummy-variables or eigenvector spatial-filtering processes may control for them as alternatives in spatial-autoregressive models of interdependence. To state up-front the conclusions reached: these analyses show (*i*) that the robust LM tests can help distinguish not just spatial-lag vs. spatial-error models but also spatial-lag from spatial group-effects as alternative processes producing observed spatial association, and (*ii*) that eigenvector spatial-filtering offers effective control for *common exposure* under the same conditions as do spatial-group dummy-variables, which is when spatial association arises by spatial group-effects or, equivalently, fully common and uniform within-group interdependence, and that this is because eigenvector spatial-filtering seems to be a subsuming generalization of spatial-group dummy-variables (something not previously noticed in the literature). The paper concludes in Section V with a proposal for an overall approach to empirical analysis of interdependence that includes as an important step using these tests and controls to provide direct answer to the question posed by *Galton's Problem*: "common exposure or contagion?"

II. Recent Developments in Spatial Interdependence vs. Common Exposure vs. Group Effects

Elhorst (2003a, 2005) derives conditional (dropping first observations) and unconditional (retaining them) likelihoods for panel data models with spatial-autoregressive lag (SAR) and random or fixed spatial group-effects. Tests of these nested model-components against each other, SAR vs. spatial group-effects, have not been derived, although given these joint-model likelihoods, L_{jm}, and those for the spatiotemporal-lag (STAR) panel-models derived by Elhorst (2003b), L_{STAR}, and for spatial group-effects (SGE) in Baltagi (2008, e.g.), L_{SGE}, Likelihood-Ratio tests of SAR or of SGE as restrictions on the joint model should be straightforward: $2ln(L_{jm}/L_{STAR}) \sim \chi_r^2$ and $2ln(L_{jm}/L_{SGE}) \sim \chi_r^2$.[4] The corresponding LM tests should also be obtainable, although they would both retain power against the spatial-error (SEM) alternative, i.e., the tests would tend to reject even if the process were truly SEM and not SAR or SGE. Robust LM-tests (Anselin et al. 1996), would adjust these standard LM tests by netting some factors related to the cross derivative of the likelihood for the model combining SAR, SGE, and SEM (see Anselin et al. 1996:79-80, and below), such that the test would not have power against the incorrect alternative in this way. However, the likelihood for a model combining SAR, SGE, and SEM has not yet been provided.

[4] The unconditional likelihoods for dynamic panel-data models with fixed or random effects require estimation of first-period effects, so they and the LR and LM tests discussed in this section use quasi-maximum-likelihood in that sense.

Baltagi et al. (2013) note that, in fact, two versions of spatial group-effects models have been introduced in the literature, and offer a generalization that subsumes them:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{X}_t\boldsymbol{\beta} + \mathbf{u}_1 + \mathbf{u}_{2t} \\ \mathbf{u}_1 &= \rho_1 \mathbf{W}\mathbf{u}_1 + \boldsymbol{\mu} \\ \mathbf{u}_2 &= \rho_2 \mathbf{W}\mathbf{u}_{2t} + \mathbf{v}_t \end{aligned} \qquad (8).$$

The specification of $\mathbf{u}_1$ entails time-invariant group-effects that exhibit (autoregressive) spatial interdependence and is attributed to Anselin (1988; Anselin et al. 2008; Baltagi et al. 2003; *inter alia*). The $\mathbf{u}_2$ specification entails time-varying group-effects exhibiting (autoregressive) spatial interdependence and is attributed to Kapoor et al. (2007) and Lee & Yu (2010,2012), *inter alia*. Baltagi et al. (2013) combines and generalizes—the generalization is to allow $\rho_1 \neq \rho_2$—these two sorts of spatial error-components (random-effects) models into (8), produces the likelihood of this generalized model under *i.i.d.*-normal $\boldsymbol{\mu}$ and $\mathbf{v}$:

$$\begin{aligned} L(\boldsymbol{\beta},\boldsymbol{\theta}) = &-\frac{NT}{2}\ln 2\pi - \frac{1}{2}\ln\det[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_v^2(\mathbf{B}'\mathbf{B})^{-1}] \\ &- \frac{T-1}{2}\ln\det[\sigma_v^2(\mathbf{B}'\mathbf{B})^{-1}] - \frac{1}{2}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta})'\boldsymbol{\Omega}_u^{-1}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}) \end{aligned} \quad \text{where } \boldsymbol{\theta}=(\sigma_v^2, \sigma_\mu^2, \rho_1, \rho_2), \; \mathbf{A} = (\mathbf{I}-\rho_1\mathbf{W}), \text{ and } \mathbf{B} = (\mathbf{I}-\rho_2\mathbf{W}) \qquad (9),$$

and derives and proves the consistency and asymptotic efficiency of Lagrange-Multiplier and Likelihood-Ratio tests of the three alternative models:

(1) H_0^A: $\rho_1 = \rho_2 = 0$, and the alternative H_1^A is that at least one component is not zero. The restricted model is the standard RE panel data model with no spatial correlation, see Baltagi (2008).
(2) H_0^B: $\rho_1 = 0$, and the alternative is H_1^B: $\rho_1 \neq 0$. The restricted model is the Anselin (1988) spatial panel model with random effects. In fact, the restricted log-likelihood function reduces to the one considered by (Anselin, 1988, p. 154).
(3) H_0^C: $\rho_1 = \rho_2 = \rho$ and the alternative is H_1^C: $\rho_1 \neq \rho_2$. The restricted model is the KKP spatial panel model with random effects.

The expressions for the LM tests—they are complex—may be found in the paper (H_0^A: p. 655, H_0^B: p. 656, and H_0^C: p. 657). In the case of a simple cross-section where T=1, however, the model and likelihood collapse to the spatial-autoregressive-error model (SEM) (7), and only H_0^A remains applicable, testing SEM versus additive spatial random group-effects. In all cases, these models and their associated tests assume the spatial dependence, in any of the three forms (four, counting the non-spatially-dependent groupwise random-effect), arises in the error term. They would therefore retain power against the spatial-autoregressive-lag (SAR) process, i.e., they would tend to reject in favor of some one, two, or all three of these spatial-error processes even if the true DGP was instead SAR, with its implied spatial-feedback and -multiplier effects of broader impact as noted above.

Notwithstanding the limitations stressed above, the possibilities for testing alternative spatial group-effects, spatial error, and spatial lag processes are obviously more promising in the panel (spatiotemporal) context than in the simple cross-section. For this more-difficult cross-sectional context, on which we focus henceforth in this paper, Anselin and Arribas-Bel (2013) show that group dummy-variables effectively control for spatial effects generally (additive or autoregressive) —something frequently claimed in some areas of applied work—only in the (one) special case that spillovers universal and uniform within groups and wholly absent across groups. I.e., if the connectivity matrix reflecting the spillover process is block diagonal by groups with off-diagonal elements of the diagonal blocks equal to $1/(n_g-1)$. In this "uniform groupwise diffusion" case (alone), autoregression with that groupwise connectivity matrix and additive group-effects (in the same grouping) are indistinguishable in a cross-section.[5] This is an instance of Manski's (1993) *Reflection Problem*: universal and uniform autoregression within a group is indistinguishable in a cross-section from a constant within the group.[6] "While little used in empirical applications, this particular weights structure… introduced in spatial econometrics in

[5] Repeated observations add modest leverage in distinguishing uniform contagion from uniform additive shock.

[6] This holds in the extreme case of a single encompassing group as well, rendering fully universal and uniform contagion indistinguishable from the constant (intercept, conditional mean) in the sample regression.

the work of Case (1991, 1992)…has been studied extensively in the context of estimation and specification testing" (Anselin and Arribas-Bel 2013:7-8). Such a *block-group* connectivity structure is a peculiar one, perhaps appropriate for interdependence arising from co-membership in certain organizations or other groups in which influence from that co-membership can be expected to emit from and be felt by all members equally, but generally less plausible than decaying, non-universal, or otherwise varying connectivity. And this peculiar "equal-weights" connectivity pattern has been shown also to manifest numerous peculiar properties, especially in cross sections.[7] "For example, Lee (2002) proved that ordinary least squares (OLS) is a consistent estimator for a spatial-lag specification with group-wise weights. Kelejian and Prucha (2002; see also Kelejian et al. 2006) showed that both OLS and two-stage least-squares (2SLS) are inconsistent in the case where only a single group is used. Smith (2009) on the other hand demonstrated the bias of maximum likelihood estimation for a model with equal weights. Estimation with near unit spatial roots are considered using equal weights in Baltagi and Liu (2010), whereas an extension to a panel data setting is given in Baltagi (2006). Problems with standard tests for spatial autocorrelation, such as the Moran's I of Cliff and Ord and the Lagrange multiplier test are demonstrated in Baltagi and Liu (2009) and Martellosio (2011)" (Anselin and Arribas-Bel 2013:8). In summary for our purposes, the relatively easy expedient of group dummy-variable controls to obtain valid estimates of other variables' effects only suffices in the case of universal, uniform spillovers within groups and no spillovers across groups,[8] which is frequently implausible and is associated with numerous peculiar properties.

III. Robust Lagrange-Multiplier Tests and Eigenvector Spatial-Filtering

[7] Again, repeated observations tend to weaken these peculiar properties.

[8] Interestingly, the potential-outcomes framework literature on causal-inference in the presence of spillovers has made much progress from within that framework only in this same peculiar case.

Anselin et al. (1996) discuss simple Lagrange-Multiplier tests (LM) for spatial-autoregressive lag versus spatial-autoregressive error models. Using the notion of models SAR (1) and SEM (7), the standard LM test-statistic for the null hypothesis $\rho = 0$, i.e., independence, against the spatial lag alternative $\rho \neq 0$ is:

$$LM_{\rho} = \frac{\hat{\sigma}_{\varepsilon}^{2}\left(\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\mathbf{y} / \hat{\sigma}_{\varepsilon}^{2}\right)^{2}}{G + T\hat{\sigma}_{\varepsilon}^{2}} \sim \chi_{1}^{2} \qquad (10),$$

where $G = (\mathbf{WX}\hat{\boldsymbol{\beta}})'(\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')(\mathbf{WX}\hat{\boldsymbol{\beta}})$ and $T = \mathrm{tr}[(\mathbf{W}' + \mathbf{W})\mathbf{W}]$. Conversely, the standard LM test-statistic of $\lambda = 0$, i.e., independence, against spatial error alternative is:

$$LM_{\lambda} = \frac{1}{T}\left(\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\hat{\boldsymbol{\varepsilon}} / \hat{\sigma}_{\varepsilon}^{2}\right)^{2} \sim \chi_{1}^{2} \qquad (11).$$

Each of these standard tests *assumes* the other spatial-autoregressive process is not present; i.e., LM_{ρ} assumes $\lambda = 0$, LM_{λ} *assumes* $\rho = 0$. The drawback of this is that each will therefore have power against the incorrect alternative; LM_{ρ} will tend to reject even if in fact the process is SEM, not SAR, and LM_{λ} will tend to reject even if the process is SAR, not SEM, which means these standard tests will not directly help in distinguishing these alternatives for specification choices. (And recall that SAR has substantively importantly different feedback and multiplier implications than SEM.)

Anselin et al. (1996)'s robust LM tests redress this shortcoming. The robust test of SEM versus independence treats ρ in the mixed SARE (spatial-autoregressive lag and error) model as a nuisance parameter, adjusting for its effect on the likelihood (intuitively, the adjustment relates to cross-derivative of the SARE likelihood with respect to λ and ρ), leaving:

$$LM_{\lambda}^{*}=\frac{\left(\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\hat{\boldsymbol{\varepsilon}}/\hat{\sigma}_{\varepsilon}^{2}-\left[T\hat{\sigma}_{\varepsilon}^{2}\left(G+T\hat{\sigma}_{\varepsilon}^{2}\right)^{-1}\right]\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\mathbf{y}/\hat{\sigma}_{\varepsilon}^{2}\right)^{2}}{T\left[1-\frac{T\hat{\sigma}_{\varepsilon}^{2}}{G+T\hat{\sigma}_{\varepsilon}^{2}}\right]}\sim\chi_{1}^{2} \qquad (12).$$

The analogously symmetric adjustment renders the test against the spatial-lag alternative robust to the presence of spatial-autoregressive-error processes.

$$LM_{\rho}^{*}=G^{-1}\hat{\sigma}_{\varepsilon}^{2}\left(\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\mathbf{y}/\hat{\sigma}_{\varepsilon}^{2}-\hat{\boldsymbol{\varepsilon}}'\mathbf{W}\hat{\boldsymbol{\varepsilon}}/\hat{\sigma}_{\varepsilon}^{2}\right)^{2}\sim\chi_{1}^{2} \qquad (13).$$

The joint LM test-statistic, finally, is simply the sum of one of the robust and one of the standard LM test-statistics:

$$LM_{\rho\lambda}=LM_{\lambda}+LM_{\rho}^{*}=LM_{\rho}+LM_{\lambda}^{*}\sim\chi_{2}^{2} \qquad (14).$$

In Monte Carlo simulations, Anselin et al. (1996) show that all five tests have the correct size in small samples. I.e., they all reject the null hypothesis at the stated rate when the null is true. The robust LM tests have lower power compared with the standard ones against the correct alternative, but the loss is relatively small and the robust tests are far less likely to reject the null against the wrong alternative.[9] The result, in sum, is a combination of tests that are highly informative for model specification in the choice of SAR or SEM specifications.

Given the demonstrations above that unobserved spatial heterogeneity will manifest as spatial error-components, and that specifically spatial group-effects will be spatial-autoregressive errors with universal and uniform spatial-weights, one important set of questions we explore in our Monte Carlo analyses below will be how well these same robust-LM tests of SAR vs. SEM

[9] For example, when the true data generating process is a spatial-error model ($\lambda\neq0$, $\rho=0$), rejection rates across the range of λ for LM_{λ} average about 5 points higher than for LM_{λ}^{*}. Meanwhile, at $\lambda=.9$, LM_{ρ} rejects in favor of the incorrect alternative (SAR) 89.9% of the time whereas LM_{ρ}^{*} rejects merely 17.1% of the time. The small power advantage of the standard compared with the large size advantage of the robust LM test is even more striking when the true DGP is SAR ($\rho\neq0$, $\lambda=0$). Rejection rates for LM_{ρ} are less than 2 points higher on average than for LM_{ρ}^{*} across the range of ρ. At $\rho=.9$, LM_{λ} rejects in favor of the incorrect alternative (SEM) 100% of the time whereas LM_{λ}^{*} rejects merely 0.6% of the time. See Anselin et al. (1996) Tables 3-6. These results are for the N=40 experiments.

can distinguish one or both of those two processes from spatial group-effects (SGE).

Griffith's spatial-filtering approach (2000, 2002, 2003, 2006), although developed and apparently addressed toward different ends, may also be of direct utility in the context of unobserved spatial heterogeneity. "Spatial filtering seeks to transform a variable containing spatial dependence into one free of it by partitioning the original georeferenced attribute variable into two synthetic variates: a spatial-filter variate capturing latent spatial dependency that otherwise would remain in the response residuals and a nonspatial-variate that is free of spatial dependence" (2006: 166). Procedurally, Griffith's proposed process begins with the eigenvector decomposition of the (centered, i.e., demeaned) connectivity matrix, **W**. These eigenvectors—some highest-eigenvalue subset of them—then "filter" the desired variable exhibiting spatial dependence simply by regressing that variable on these eigenvectors of **W**. Recalling that the set of K eigenvectors for a rank(K) matrix reorganizes all of the same information in the original matrix into K orthogonal components, arrayed by share of that information represented from highest-eigenvalue eigenvector to lowest, and that the bias from omitting spatially associated unobservables will be proportionate to how similar their pattern of incidence is to the **Wy** or **We** included in the model, including those eigenvectors as controls in the regression would seem a very strong manner of controlling for unobserved heterogeneity in seeking good estimates of interdependence strength. We explore this conjecture in our Monte Carlo Analyses below, finding good support for it, and discovering that, in fact, Griffith's eigenvector spatial-filtering generalizes and subsumes the group dummy-variable ad hoc strategy often deployed in the literature.

IV. Monte Carlo Analyses

In our simulations, we explore the possibility of distinguishing, in cross-sections of data, spatial group-effects from interdependence within spatial groups of both the spatial (AR) lag and spatial error variety. Thus, we have three data generating processes. The group effects model is

$$\mathbf{y}_{ig} = \mathbf{x}_{ig}\boldsymbol{\beta} + \boldsymbol{\eta}_g + \varepsilon_{ig}, \tag{15}$$

where $\mathbf{y}_{ig}, \mathbf{x}_{ig}$, and ε_{ig}, the outcome, covariates and disturbance respectively, refer to unit or individual *i* in group *g*, and $\boldsymbol{\eta}_g$ is the common group effect. The SEM model is

$$\mathbf{y}_{ig} = \mathbf{x}_{ig}\boldsymbol{\beta} + (I - \lambda\mathbf{W})^{-1}\varepsilon_{ig}, \tag{16}$$

and the SAR model is

$$\mathbf{y}_{ig} = (I - \lambda\mathbf{W})^{-1}(\mathbf{x}_{ig}\boldsymbol{\beta} + \varepsilon_{ig}), \tag{17}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{W}_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathbf{W}_G \end{bmatrix}, \tag{18}$$

the complete weights matrix, has a block diagonal structure for G groups, when the units or individuals in the sample are stacked by groups. The group-effects and intra-group interdependence models are indistinguishable when all of the off-diagonal elements of each group connectivity matrix, $\mathbf{W}_g$, are $1/(n_g - 1)$. In this case, the weights are both uniform and universal. Every unit or individual in the group is connected to every other member of the group, and the strength of the connections is uniform.

To make the two models empirically distinct, we vary the degree of intra-group connectivity. At the high end, we set the degree of intra-group connectivity at 80%—each member of the group is connected to 80% of the other members (randomly selected)—and at the low end, we set connectivity at 20%, $c \in \{20\%, 80\%\}$. We assume the connectivity weights are uniform and sum to one. That is, the weights are $1/n_c$, where n_c is the number of intra-group connections. We focus on a relatively modest sample size where $n_g = 20$ and $G = 4$. For both our low and high connectivity cases, we estimate three regressions using OLS: a simple regression of ***y*** on ***x***, a regression of ***y*** on ***x*** and a set of group dummy variables, and a regression of ***y*** on ***x*** and a set of filtering eigenvectors (Griffith, 2000, 2002, 2003, 2006). The last two regressions incorporate strategies to address spatial clustering on unobservables. The first, spatial dummy variables, is very common in the literature. The second, spatial filtering, is employed less frequently.

We use the OLS residuals to calculate Anselin et al.'s (1996) robust Lagrange-Multiplier tests. The main advantage of these tests is that they have low power against the incorrect alternative. Speficially, the robust LM test for the spatial lag model has low power against the spatial error model when it is the true data generating process and vice versa. We are interested in the degree to which both these tests have power against the spatial group effects model as well as the extent to which strategies for addressing clustering on unobservables, group dummy variables and spatial filtering, reduce our power to detect spatial error and spatial lag interdependence. To facilitate comparability with the Monte Carlo results in Anselin et al., we follow their experimental design when possible. The disturbances, $\boldsymbol{\varepsilon}$, are standard normal variates, and $\mathbf{x}_{ig}$ is a row vector consisting of a constant term and two variates drawn from a uniform (0,10) distribution. The column vector of coefficients contains ones, $\boldsymbol{\beta} = \mathbf{1}$, and $\lambda = 0.9$ (strong interdependence). The critical difference between our experiments and Anselin et al. is the spatial weights matrix.

Before presenting the Monte Carlo results, we note an important connection between the group dummy and eigenvector filter approaches to clustering on unobservables that, to our knowledge, has not received attention in the spatial econometrics or statistics literatures. If a weights matrix $\mathbf{W}$ only allows for connections within groups, then the dummy variable and eigenvector filter approaches are equivalent when the number of eigenvectors is *G*-1. For example, if *G*=4, regardless of the number of group members (n_g) and degree of connectivity (c), the first three eigenvectors (i.e., the eigenvectors with the largest associated eigenvalues) from the decomposition of $\mathbf{W}$ give fixed values for every member of the four groups, so the regression estimates and the LM tests are exactly the same whether you include these eigenvectors or group dummy variables. Given this equivalence, we include *G* eigenvectors in our spatially filtered regressions. In general, adding eigenvectors beyond *G*-1 does not seem to improve the performance of the specification tests. The main advantage of the eigenvector approach over dummy variables is that one does not need to have *a priori* knowledge of the relevant spatial groupings, only the dyadic level connections, which is helpful when the boundaries between groups are not as clear as they are in our Monte Carlo experiments.

The results are reported in Tables 1-2. We start with the low connectivity case (20%) in Table 1. The entries refer to the sampling distributions of the two robust LM statistics from Anselin et al. (1996). The first entry in each cell is the value of the statistic at either the 90, 95, 99th percentiles of the sampling distribution. The second entry in parentheses is the percentage of the sampling distribution that is greater than the relevant χ_1^2 critical value (i.e., empirical rejection percentages). For example, with respect to the first estimation strategy (no group dummies), the value at the 90th percentile of the robust LM sampling distribution for the spatial error alternative when the DGP is the spatial fixed effects model is 59.99 and 85.8% of the values from this distribution are greater than the critical value χ_1^2 , 2.71.

[Tables 1 & 2 About Here]

The robust spatial error LM test has substantial power against the spatial group effects model (see the upper third of Table 1). One frequent interpretation of the spatial error model is that it captures spatial clustering on unobservables, rather than "true" interdependence in the error term. Our results suggest that this is a very plausible interpretation. Group dummies and spatial filtering reduce the power of this test dramatically, though not quite to the levels one would expect based on the χ_1^2 distribution. Nevertheless, group dummies and spatial filtering do seem to provide significant protection against rejecting the null hypothesis in favor of the incorrect alternative (i.e., the spatial error model). The empirical rejection frequencies at the 95th percentile when group dummies and spatial filtering are used are 8.1% and 10.1% respectively compared to 82.7% when there is no attempt to account for potential spatial-group fixed effects.

The power of the robust spatial error LM test against the spatial error model is very high, though not quite as high as in Anselin et al (see the middle third of Table 1). The empirical rejection rate at p=0.05 critical value in our experiments is 94% compared to rates near 100% in Anselin et al. Note that when the DGP is a spatial error model with low connectivity, the costs of including group dummies and eigenvectors is nontrivial. The empirical rejection rate drops from 94% to just above 30%.

The robust spatial error LM test also has power against the spatial (AR) lag model (see the bottom third of Table 1). We find rejection rates that are similar to those reported in Anselin et al. for their N=81, queen contiguity case: 53.7% vs. 35.1% (see Anselin et al. 1996, Table 6, p. 96). Group dummies and spatial filtering reduce the rejection rates to 21.1% and 19.9%. For our strong interdependence DGP, $\lambda = 0.9$, the robust spatial lag LM test never fails to reject the null hypothesis, even when group dummies and spatial eigenvectors are added to the regression model. Thus, these filters provide some protection against rejecting the null hypothesis in favor of the incorrect alternative (spatial error) at no cost in terms of power with respect to the correct alternative (spatial lag).

The results for the high connectivity case (80%) are presented in Table 2. Not surprisingly, the power of the robust spatial error LM test increases against the incorrect spatial group effects model relative to the low connectivity case (see upper third of Table 2). Remember, at 100% connectivity, the two models are indistinguishable. At the p=0.05 critical value, the rejection rate increases from 82.7% to 90.1%. At high levels of connectivity, the power of the robust spatial error LM test against the spatial error model increases as well from 94% to 96.5% at p=0.05 (see middle third of Table 2). In this case, high connectivity, including group dummies and eigenvectors comes at a very high cost, reducing the power of the LM test against the true spatial error alternative. In fact, rejection rates are similar to what one would expect if the null hypothesis were true, suggesting that all evidence of interdependence is removed from the estimated residuals. At high levels of connectivity, the robust spatial lag LM test has unacceptably high power against both the spatial group effects and spatial error models. This contrasts sharply with the low connectivity Monte Carlo results of Anselin et al. (1996). With respect to the spatial error DGP, our rejection rate at the p=0.05 critical value is 55.9%, which compares to an average rejection rate of 23.3% in the Anselin et al. experiments. In our high connectivity experiments, each unit is connected to 15 other units, whereas in Anselin et al. the average number of connections across their four weights matrices are 4.2, 4.7, 3.6, and 6.7. Including group dummies and spatial eigenvectors in the regression model does reduce the rejection rates to levels consistent with what we would expect under the null hypothesis. The one case in our experiments where spatial-group dummies and spatial-filtering eigenvectors fail to provide protection against an incorrect alternative is with the high connectivity, strong interdependence spatial (AR) lag DGP (see bottom third of Table 2). Rejection rates for the robust spatial error LM test at p=0.05 are above 90%.

V. Conclusion

One of the central challenges to empirical inference in the context of potentially interdependent observations, known as *Galton's Problem*, is the difficulty distinguishing spatial correlation in outcomes due to the units' exposure to spatially correlated shocks ('common exposure') from spatially dependent outcomes due to interdependence ('spillovers' or 'contagion') among units. The applied researcher's first, and until recently only, defense against confusing these substantively importantly different processes empirically has been to control as best possible with observable regressors and/or groupwise dummy-variables for correlated-shocks processes when estimating interdependence models (spatial autoregression). Specifying empirical models and measures as precisely and powerfully as possible remains the optimal practice, but these strategies cannot guard fully against the possibility of exposure to *unobserved* exogenous shocks that are distributed spatially in some manner not fully common to some set of units (which fixed or random group effects could account) or fully controlled by observable exogenous factors (control variables), but are instead distributed across units more similarly to the pattern by which the outcome is contagious.

Our results suggest that spatial filtering via eigenvector decomposition of the weights matrix is a sensible strategy to address spatial clustering on unobservables. In fact, eigenvector spatial-filtering seems to be a generalization that subsumes spatial dummy-variables, something not previously noticed in the literature. Spatial filtering provides protection against rejecting the null hypothesis of independent outcomes in favor of incorrect interdependence alternatives. Moreover, when there is interdependence, the cost of spatial filtering in terms of lost power is relatively modest, though, in our experiments, increasing with the degree of connectivity.

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Table 1. Monte Carlo Experiments: Intra-group Connectivity 20% (N20, G4)

DGP: Spatial Fixed Effects	Without Group Dummies	With Group Dummies	With Spatial Filtering
Spatial Error			
χ_1^2 (90%): 2.71	59.99 (85.8%)	3.46 (15.3%)	3.86 (17.4%)
χ_1^2 (95%): 3.84	71.69 (82.7%)	4.71 (8.1%)	5.16 (10.1%)
χ_1^2 (99%): 6.63	93.24 (75.9%)	7.76 (1.7%)	8.25 (2.3%)
Spatial Lag			
χ_1^2 (90%): 2.71	5.24 (23.9%)	2.87 (11.2%)	2.92 (11.4%)
χ_1^2 (95%): 3.84	7.28 (16.1%)	4.05 (5.6%)	3.99 (5.7%)
χ_1^2 (99%): 6.63	12.63 (6.2%)	6.97 (1.2%)	7.18 (1.2%)
DGP: Spatial Error Model			
Spatial Error			
χ_1^2 (90%): 2.71	58.44 (96.5%)	6.94 (40.3%)	7.02 (44.6%)
χ_1^2 (95%): 3.84	69.63 (94.0%)	8.86 (30.6%)	8.87 (31.2%)
χ_1^2 (99%): 6.63	89.52 (87.2%)	12.97 (11.4%)	13.19 (11.6%)
Spatial Lag			
χ_1^2 (90%): 2.71	4.65 (22.1%)	3.20 (12.9%)	3.23 (12.9%)
χ_1^2 (95%): 3.84	6.46 (14.1%)	4.35 (6.7%)	4.44 (6.9%)
χ_1^2 (99%): 6.63	10.65 (4.6%)	7.26 (1.6%)	7.34 (1.5%)
DGP: Spatial Lag Model			
Spatial Error			
χ_1^2 (90%): 2.71	10.16 (63.3%)	5.41 (35.6%)	5.26 (34.0%)
χ_1^2 (95%): 3.84	11.69 (53.7%)	6.80 (21.1%)	6.58 (19.9%)
χ_1^2 (99%): 6.63	14.93 (30.6%)	9.65 (5.4%)	9.53 (4.9%)
Spatial Lag			
χ_1^2 (90%): 2.71	53.85 (100%)	54.66 (100%)	54.65 (12.9%)
χ_1^2 (95%): 3.84	55.63 (100%)	56.45 (100%)	56.41 (6.9%)
χ_1^2 (99%): 6.63	58.34 (100%)	59.36 (100%)	59.38 (1.5%)

Table 2. Monte Carlo Experiments: Intra-group Connectivity 80% (N20, G4)

DGP: Spatial Fixed Effects	Without Group Dummies	With Group Dummies	With Spatial Filtering
Spatial Error			
χ_1^2 (90%): 2.71	180.02 (91.3%)	3.83 (21.4%)	3.73 (19.5%)
χ_1^2 (95%): 3.84	216.08 (90.1%)	4.81 (10.0%)	4.72 (9.5%)
χ_1^2 (99%): 6.63	278.86 (87.2%)	7.22 (1.5%)	6.98 (1.2%)
Spatial Lag			
χ_1^2 (90%): 2.71	17.89 (56.1%)	2.95 (11.5%)	3.04 (11.7%)
χ_1^2 (95%): 3.84	23.06 (48.8%)	4.25 (6.2%)	4.32 (6.5%)
χ_1^2 (99%): 6.63	32.33 (36.1%)	7.18 (1.4%)	7.47 (1.6%)
DGP: Spatial Error Model			
Spatial Error			
χ_1^2 (90%): 2.71	263.29 (97.3%)	2.74 (10.3%)	2.69 (9.9%)
χ_1^2 (95%): 3.84	297.90 (96.5%)	3.65 (4.3%)	3.60 (4.3%)
χ_1^2 (99%): 6.63	356.52 (95.3%)	5.37 (.34%)	5.45 (.28%)
Spatial Lag			
χ_1^2 (90%): 2.71	23.34 (62.9%)	3.04 (12.4%)	3.07 (12.5%)
χ_1^2 (95%): 3.84	30.01 (55.9%)	4.31 (6.5%)	4.43 (6.6%)
χ_1^2 (99%): 6.63	41.01 (43.3%)	6.99 (1.4%)	7.22 (1.6%)
DGP: Spatial Lag Model			
Spatial Error			
χ_1^2 (90%): 2.71	91.93 (99.7%)	13.37 (97.8%)	13.25 (97.5%)
χ_1^2 (95%): 3.84	100.32 (99.7%)	14.84 (93.4%)	14.64 (93.4%)
χ_1^2 (99%): 6.63	115.95 (99.6%)	18.08 (71.7%)	18.02 (70.2%)
Spatial Lag			
χ_1^2 (90%): 2.71	67.51 (99.7%)	22.33 (98.2%)	22.19 (97.8%)
χ_1^2 (95%): 3.84	69.77 (99.7%)	25.08 (96.0%)	25.25 (95.8%)
χ_1^2 (99%): 6.63	72.48 (99.3%)	30.46 (88.2%)	30.64 (87.6%)