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Continuous Time Scheduling for Sugarcane Harvest Logistics in Louisiana

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Despite a growing global appetite for sugar as both a foodstuff and a fuel source, there exists limited literature that explores sugarcane operations. In this paper, we look at the scheduling harvest and logistics operations in the state of Louisiana in the United States. These operations account for significant portions of the total sugarcane production costs. We develop an integer-programming model for coordinating harvest and transport of sugarcane. The model seeks to reduce vehicle waiting time at the mill by maximizing the minimum gap between two successive arrivals at the mill. To help improve tractability, we introduce valid inequalities and optimality cuts. We also demonstrate how to adapt solutions from a previous discrete-time model. Our results show that arrivals can easily be coordinated to reduce truck waiting time at the mill.

Keywords: Logistics, scheduling, sugarcane, integer programming

1. Introduction

With a growing global appetite for sugar as both a foodstuff and a fuel source (McConnell, Dohlman, and Haley 2010; Valdes 2011; Wexler December 17, 2012; Foreign Agricultural Service 2014), the importance of efficient and effective sugarcane harvests logistics has never been higher. In this paper, we look at sugarcane harvest operations in Louisiana, a state in the United States. Sugarcane harvests in the Louisiana have three operations that must be coordinated: infield operations, over-the-road transport, and mill operations. Infield operations usually occur in several pre-specified farms and have numerous components. First, the cane is cut in the field, usually using a mechanical harvester that cuts the cane into uniformly sized billets (12-18 inches). While in operation, the harvester continuously feeds billets into a cart pulled by an infield transporter. This infield transporter and cart combination runs alongside the harvester, and, when the cart is filled, the transporter and cart combination must be rotated with another infield vehicle and its associated cart for continuous harvest operations. Filled carts are transported to a loading pad that serves the farm. At the loading pad, the sugarcane is transferred to trucks that take the harvested cane from the farms to the mill. The final operation of the harvest takes place at the mill where the trucks are unloaded. Once a truck is unloaded, it can return to a farm for its next load.

Harvest operations on farms are generally conducted only during daylight hours, and most farms begin harvesting operations as early in the morning as possible. One of the key challenges in both countries is the lack of coordination among growers as well as between growers and the mill. For example, according to last census, there are 473 operating sugarcane farms.As a result, there can be a long queue of trucks waiting to be unloaded at the mill yard. This extra waiting time at the mill reduces the number of loads that can be hauled by each individual truck. Thus, the existing harvest and transport arrangement increases the number of trucks required to haul the mill's daily

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quota of sugarcane. Collaboration between farmers on the one hand and the mill on the other could improve the overall efficiency of harvested cane transport operations by reducing number of trucks required to haul the cut cane.

In this paper, we seek to reduce congestion at the mill and as well as the number of trucks required to serve the harvest. We seek to reduce mill congestion rather than to model the trucks directly because the latter leads to intractable models. We reduce congestion by seeking to maximize the time between the arrivals of loads to the mill. This objective has the effect of minimizing congestion or queueing at the mill because it maximizes the average interarrival time of loads to the mill and thus minimizes the utilization of the unloading operation at the mill. As we demonstrate in our computational results, our objective also reduces variance in inter arrival times. It is well known in the queueing literature that reducing utilization and variation in interarrival times reduces queueing.

We consider a set of fields which provide a pre-specified set of loads to the mill. The farms harvest at a fixed rate. All the trucks start their shifts at the mill. The travel time between the farms and the mill is deterministic. The trucks arriving at the mill form a single first in first out queue. When a truck is unloaded, it is available for the next dispatch. The cycle continues until all the loads are picked up from the farms are unloaded at the mill. Our objective is to maximize the minimum time between consecutive truck arrivals to the mill. The objective is maximized by setting the start times of the harvests at the farms. Given the solution to the math program, we generate truck assignments.

This paper makes two contributions to the literature. We demonstrate that, by spreading the harvesting throughout the daylight hours, the mills and the growers can achieve significant savings. We show that we can achieve this savings by coordinating start times at the fields. Through start time coordination, we spread arrivals of trucks, reducing congestion, and thus reducing the number of trucks required to serve the harvest. Our computational results show that setting the start times of harvests at the various farms is sufficient to achieve the necessary coordination. These validate the conjecture in Salassi and Barker (2008) that truck congestion at the mill could be reduced by coordinating the start times of the harvests at the farms. Second, we introduce a model that eliminates the discretization required in Salassi and Barker (2008) and in Lamsal, Jones, and Thomas (2013). We demonstrate that eliminating discretization reduces the number of trucks. We also introduce a series of valid inequalities that lead to a tractable model. As a minor contribution, we demonstrate the value of using the discrete-time model presented in Lamsal, Jones, and Thomas (2013) to generate initial feasible solutions.

Section 2 of the paper discusses previous work on sugarcane logistics. Section 3 presents our model as well as valid inequalities and optimality cuts. In Section 4, we describe the solution approach. In Section 5, we present the results of acomputational study using our model. The study uses the benchmark problems developed by Lamsal, Jones, and Thomas (2015). These benchmark datasets use publicly available data on the geographical locations of each of Louisiana's 456 sugarcane farms and 11 sugarcane mills as well as their production and processing rates to construct a set of 11 sugarcane logistics problems (one for each of the 11 mills in Louisiana). Section 6 presents our conclusions.

2. Literature Review

Through the years, a number of authors have sought to optimize various aspects of the sugarcane supply chain. However, the infrastructures vary from country to country in ways that make models suitable for one country not suitable for others. Of note, the sugarcane harvesting and transport in different countries have varying divisions of decision making between farm and mill levels. In general, a lack of coordination among the decision makers affects the efficiency of the whole system. For a more detailed discussion of sugarcane harvest logistics and the literature related to the infrastructure different from that discussed in this paper, we refer the reader to Lamsal, Jones, and Thomas (2015). The most recent work on sugarcane focuses on determining what farms to harvest on what days and ignores the operational considerations involved in the transportation of sugarcane (Jena and Poggi 2013; Sethanan, Theerakulpisut, and Neungmatcha 2014).

Most closely related to the work in this paper are Salassi et al. (2009) and Lamsal, Jones, and Thomas (2013). Both papers use mixed integer mathematical programming models to evaluate the impact of alternative harvest schedules at the farms that result in shorter queues at the mill of the trucks waiting to be unloaded thus reducing the total truck hours and the number of trucks needed to haul the cane. Salassi et al. (2009) and Lamsal, Jones, and Thomas (2013) divide the day into blocks of time and use discretization techniques to spread arrivals among these blocks of time. Lamsal, Jones, and Thomas (2013) show that as the time blocks become smaller, the model produces more desirable results, in the sense that the loads arrivals are spread more uniformly throughout the day and also require fewer trucks. On the flip side, the complexity of the problem increases when the size of the time blocks decreases, eventually leading to a computationally intractable problem. Our objective is motivated by the results in Lamsal, Jones, and Thomas (2013). In this paper, we make the problem continuous by removing the notion of time blocks and maximally spread the load arrivals by maximizing the smallest gap between two successive arrivals at the mill.

Also related to the work in this paper is Higgins et al. (2004) and Higgins and Laredo (2006). The two papers develop a framework for integrating a complex harvesting and transportation system for sugar production in Australia. They seek to reduce the congestion at the mill. They use heuristic methods to produce transportation schedules such that mill idle time, queue length and the number of trucks needed to haul the cut cane are reduced. We add to their work by coordinating harvest schedules with the transport schedules to further reduce the queue length and number of trucks needed.

Lamsal, Jones, and Thomas (2015) is also related to the work in this paper. However, Lamsal, Jones, and Thomas (2015) focuses on sugarcane operations in Brazil where the infrastructure, notably the level of vertical integration, differs from that in the US and Australia. Consequently, Lamsal, Jones, and Thomas (2015) develop a model that coordinates the load arrivals at the mill with the objective of minimizing what is known as 'cut-to-crush delay,' the time between when a stalk of sugarcane is cut and when it is processed at the mill.

While not optimization models, Singh and Pathak (1994) and Arjona, Bueno, and Salazar (2001) develop simulation models for Thailand and Mexico, respectively, that determine the cost of given harvest scenarios. In particular, the models are capable of trading off the cost and performance of using particular number of trucks to transport cut sugarcane from the field to the mill. Much older work by Whitney and Cochran (1976) seeks to use queuing theory to predict delivery rates of the harvested cane.

3. Model

In this section, we present a formal model for determining the start times of the harvests at each farm. We also present valid inequalities and an optimality cut that strengthen the model. This model and its solution are the first phase of our solution approach. Our overall goal is to minimize the number of trucks required to pick up loads at the times when they become ready. However, our objective maximizes the minimum time between arrivals to the mill. As noted earlier, directly modeling the minimization of the number of trucks results in an intractable problem. As discussed in the Introduction and as our results will show, our choice of objective, when coupled with our second phase, is capable of reducing the trucks needed to serve the harvest. We discuss the second phase problem in Section 4.

Our model assumes that we know the farms that will provide loads to the mill, the amount of time required to harvest a load at each of the farms, and the travel time from each farm to the mill. Further, the model assumes that we know the number of loads to be produced by the mill for the day. These needs are determined by the amount of sugarcane that is to be crushed for the day and the processing rate of the mill. We assume that the number of loads are determined by an exogenous decision maker. The number of loads allocated to each farm is reflected in the parameter n_i.

Next, we introduce the notation and a basic model and discuss the constraints.

3.1 *Base Model*

Sets:

F set of farms.

Parameters:

h_i	$i \in F$	time to harvest one load at the farm i
t_i	$i \in F$	travel time from mill to the farm i
n_i	$i \in F$	daily load quota of the farm i
$U_{iji'j'}$	$i, i' \in F, j \in \{1 \dots n_i - 1\}, j' \in \{1 \dots n'_i - 1\}, i < i'$	upper bound to the difference of x_{ij} and $x_{i'j'}$
$L_{iji'j'}$	$i, i' \in F, j \in \{1 \dots n_i - 1\}, j' \in \{1 \dots n'_i - 1\}, i < i'$	lower bound to the difference of x_{ij} and $x_{i'j'}$.

Variables:

y_i	$i \in F$	time when harvesting starts at farm i
z_{ij}	$i \in F, j \in \{1 \dots n_j\}$	ready time or time that load j is ready for pick-up from farm i
x_{ij}	$i \in F, j \in \{1 \dots n_j\}$	arrival time at the mill for load j from farm i
$S^+_{iji'j'}$	$i, i' \in F, j \in \{1 \dots n_i\}$, $j' \in \{1 \dots n_{i'}\}, i < i'$	dummy variable that takes the value as the difference between x_{ij} and $x_{i'j'}$ if $x_{ij} > x_{i'j'}$ and zero otherwise
$S^-_{iji'j'}$	$i, i' \in F, j \in \{1 \dots n_i\}$, $j' \in \{1 \dots n_{i'}\}, i < i'$	dummy variable that takes the value as the difference between x_{ij} and $x_{i'j'}$ if $x_{ij} < x_{i'j'}$ and zero otherwise
$B_{iji'j'}$	$\in \{0,1\}$ $i, i' \in F$, $j \in \{1 \dots n_i\}$, $j' \in \{1 \dots n_{i'}\}, i < i'$	binary variable that takes the value of 1 if x_{ij} is larger than $x_{i'j'}$ and 0 if x_{ij} is smaller than $x_{i'j'}$
Obj		objective value, minimum gap between two consecutive arrivals.

Objective:

$$\max \quad Obj$$

Constraints:

$$z_{ij} = y_i + j \times h_i \qquad \forall (i,j) \mid i \in F,\ j \in \{1 \ldots n_i\} \tag{1}$$

$$x_{ij} = z_{ij} + t_i \qquad \forall (i,j) \mid i \in F,\ j \in \{1 \ldots n_i\} \tag{2}$$

$$x_{ij} - x_{i'j'} = S^{+}_{iji'j'} - S^{-}_{iji'j'} \qquad \forall (i,j),(i',j') \mid i,i' \in F,\ j \in \{1 \ldots n_i\}, j' \in \{1 \ldots n_{i'}\}, i < i' \tag{3}$$

$$0 \le S^{+}_{iji'j'} \le U_{iji'j'} \cdot B_{iji'j'} \qquad \forall (i,j),(i',j') \mid i,i' \in F,\ j \in \{1 \ldots n_i\}, j' \in \{1 \ldots n_{i'}\}, i < i' \tag{4}$$

$$0 \le S^{-}_{iji'j'} \le |L_{iji'j'}| \times (1 - B_{iji'j'}) \qquad \forall (i,j),(i',j') \mid i,i' \in F,\ j \in \{1 \ldots n_i\}, j' \in \{1 \ldots n_{i'}\}, i < i' \tag{5}$$

$$Obj \le S^{+}_{iji'j'} + S^{-}_{iji'j'} \qquad \forall (i,j),(i',j') \mid i,i' \in F,\ j \in \{1 \ldots n_i\}, j' \in \{1 \ldots n_{i'}\}, i < i'. \tag{6}$$

Constraints (1) relate the harvest start times of the farms to the ready times of all the loads from the respective farms. Constraints (2) relate the ready times of the loads with the loads' arrival times at the mill. In the case of the Louisiana instances, the equality in this constraint reflects that loads are required to be picked up from the farms as soon as the harvesting of the load has been completed. We note that the return time to the mill t_i can be thought of as included any time that is required to load the truck's trailer and prepare for trailer. There is no need to model this time separately.

Contraints (3) represent the difference between two arrival times as the difference of two non-negative variables. We note that, for two arbitrary arrival times x_{ij} and $x_{i'j'}$ and $i' < i$, $x_{ij} - x_{i'j'} = -(x_{i'j'} - x_{ij})$ and $S^{+}_{iji'j'} - S^{-}_{iji'j'} = -\left(S^{+}_{i'j'ij} - S^{-}_{i'j'ij}\right)$. Further, we do not define Contraints (3) for $i' \le i$. Such constraints are unnecessary. In addition, we do not consider the situation when $i = i'$ because the difference between two closest arrivals from the same farm is fixed.

Constraints (4) and Constraints (5) force one of the two non-negative variables from Constraints (3) to be zero. Unlike in min-max formulations, in the max-min objective, increasing $S^{+}_{iji'j'}$ or $S^{-}_{iji'j'}$ improves the objective value. Thus, we need to introduce constraints to force one of the variables in each pair to be zero. The variable $S^{+}_{iji'j'}$ is positive and $S^{-}_{iji'j'}$ is zero if x_{ij} is larger than $x_{i'j'}$, and if x_{ij} is smaller than $x_{i'j'}$, $S^{-}_{iji'j'}$ is positive and $S^{+}_{iji'j'}$ is zero. The binary variable $B_{iji'j'}$ takes the value 1 when x_{ij} is larger than $x_{i'j'}$ and zero when x_{ij} is smaller than $x_{i'j'}$. Constraints (6) forces the objective to be larger than the absolute difference of any two arrivals.

We note that, as in Lamsal, Jones, and Thomas (2013), this model does not require the truck counting constraints found in Salassi and Barker (2008) (Constraints (5) in Salassi and Barker (2008)). Rather, we determine the required number of trucks in a subsequent phase. Our procedure is discussed in Section 4.

3.2 *Valid inequalities and Optimality Cuts*

In this section, we present results that strengthen the present formulation. We first note that we can bound the arrival of loads from each farm to the mill using simple constraint propagations. Suppose a farm is 10 minutes away from the mill, and the time to harvest a load at that farm is 30 minutes. Also, suppose the front needs to produce 20 loads. If the harvesting at the front can start at 6:00 am and must finish by 6:00 pm, the bounds for arrival at the mill for the first load from the front are [6:40 am, 8:40 am]. The lower bound is 6:40 am because, if harvesting starts at 6:00 am, the first load arrives at the mill at 6:40 am. The upper bound is 8:40 am because, if the load is not ready by 8:30 am (thus, making the arrival time 8:40 am), the 20^{th} load cannot be completed by 6:00 pm. The analogies can be drawn with earliest finish and latest finish calculations used in critical path analysis. These bounds are fairly tight in our instances. To implement this constraint propagation, we add the following constraints to the model:

$$a_{ij} \leq x_{ij} \leq b_{ij} \qquad\qquad \forall (i,j) \mid i \in F,\ j \in \{1 \ldots n_i\},$$

where a_{ij} and b_{ij} are the bounds for the j^{th} load from farm i.

Next, we state and prove a proposition demonstrating monotonicity among the binary variables B. The result takes advantage of the fact that all loads from any given farm must be picked up at their ready time and the physical constraint of the harvest time for each load.

Proposition 3.1 (Monotonicity). *For all $i, i', j,$ and j' such that $i, i' \in F$, $i < i'$, $j \in 1, \ldots, n_i - 1$, and $j' \in 1, \ldots, n_{i'}$*

$$B_{iji'j'} \leq B_{i(j+1)i'j'}.$$

Similarly, for all $i, i', j,$ and j' such that $i, i' \in F$, $i < i'$, $j \in 1, \ldots, n_i$, and $j' \in 1, \ldots, n_{i'} - 1$

$$B_{iji'j'} \geq B_{iji'(j'+1)}.$$

Proof. Consider a series of arrivals from farm i, $x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}$. By Constraints (1) and (2), we know that $x_{i,1} < x_{i,2} < \cdots < x_{i,n_i}$. Next, consider any load from farm i'. Let this be load j'. The arrival time for the j'^{th} load from farm i' is $x_{i',j'}$. Subtracting the arrival time $x_{i',j'}$ from the arrival times of each of the loads from farm i gives us $(x_{i,1} - x_{i',j'}) < (x_{i,2} - x_{i',j'}) < \cdots < (x_{i,n_i} - x_{i',j'})$.

As a result of Constraints (4) and (5), for any load j from farm i, $B_{i,j,i',j'} = 1$ if $x_{i,j} - x_{i',j'}$ is positive and 0 otherwise. Then, because $(x_{i,j} - x_{i',j'}) < (x_{i,j+1} - x_{i',j'})$ for every $j \in \{1, \ldots, n_i - 1\}$, $B_{iji'j'} \leq B_{i(j+1)i'j'}$.

The second part of the proof follows analogously. Again as a result of Constraints (1) and (2), we have the following series of inequalities $(x_{i,j} - x_{i',1}) > (x_{i,j} - x_{i',2}) > \cdots > (x_{i,j} - x_{i',n_{i'}})$, which implies $B_{i,j,i',1} \geq B_{i,j,i',2} \geq \cdots \geq B_{i,j,i',n_i'}$. □

As a result of Proposition 3.1, we add following sets of valid inequalities to the base model:

$$B_{iji'j'} \leq B_{i(j+1)i'j'}\ \forall (i,j),(i',j') \mid i,i' \in F, j \in \{1 \ldots n_i - 1\}, j' \in \{1 \ldots n_{i'}\}, i < i' \tag{7}$$

$$B_{iji'j'} \geq B_{iji'(j'+1)}\ \forall (i,j),(i',j') \mid i,i' \in F, j \in \{1 \ldots n_i\}, j' \in \{1 \ldots n_{i'} - 1\}, i < i'. \tag{8}$$

We next present two optimality cuts that use the value of a feasible solution to bound the number of arrivals to the mill that can occur between to successive arrivals from a given farm. The first result bounds the number of arrivals that occur from a single farm in the interval between two successive arrivals from another. The second result bounds the number of arrivals from all farms that can occur in the interval between two successive arrivals from any farm. In both cases, we take advantage of the objective value of a feasible solution and also the fact that the harvest rates at each farm are constant and that we require loads to be picked up when they are ready.

Proposition 3.2. *Given a feasible solution value $\underline{obj}$ and for two successive loads arriving to the mill from farm i, the number of maximum arrivals originating from any farm $i' \neq i$ is bounded by $\left(\left\lfloor \frac{h_i - 2\times \underline{obj}}{h_{i'}} \right\rfloor + 1\right)$.*

Proof. Let $x_{i,j}$ and $x_{i,j+1}$ be any two successive arrivals to the mill from the farm i . By construction, we know that $x_{i,j+1} - x_{i,j} = h_i$. Further, the time between any two arrivals must also be greater than the given objective value of a feasible solution $\underline{obj}$. Then, there exists at most $h_i - 2 \times \underline{obj}$ units of time in which loads can arrive. We also know from the data that farm i' produces a load every $h_{i'}$ time units and thus all arrivals from farm i' are separated by at least $h_{i'}$.

Thus, if $\frac{h_i - 2\times\underline{obj}}{h_{i'}}$ is non-integer, no more than $\left(\left\lceil\frac{h_i - 2\times\underline{obj}}{h_{i'}}\right\rceil\right)$ loads can arrive from farm i' between two successive loads from farm i. However, if $\frac{h_i - 2\times\underline{obj}}{h_{i'}}$ is integer, we must account for the fact that a load can arrive exactly at time $x_{i,j} + \underline{obj}$ and resultantly the bound becomes $\left(\left\lceil\frac{h_i - 2\times\underline{obj}}{h_{i'}}\right\rceil + 1\right)$. However, this bound is not tight in the non-integer case. We can tighten the bound by instead using $\left(\left\lfloor\frac{h_i - 2\times\underline{obj}}{h_{i'}}\right\rfloor + 1\right)$. □

To introduce inequalities that take advantage of the result in Proposition 3.2, we first note that $\sum_{j'\in 1..n_{i'}} B_{i(j+1)i'j'}$ counts the total number of arrivals prior to $x_{i,j+1}$ from farm i'. Similarly, the term $\sum_{j'\in 1..n_{i'}} B_{iji'j'}$ counts the number of arrivals prior to $x_{i,j}$ from farm i'. Thus, the sum

$$\left(\sum_{j'\in 1..n_{i'}} B_{i(j+1)i'j'} - \sum_{j'\in 1..n_{i'}} B_{iji'j'}\right)$$

reflects the total number of loads from farm i' that arrive to the mill between $x_{i,j}$ and $x_{i,j+1}$. Thus, as a result of Proposition 3.2 and when a feasible solution exists, we add the following set of optimality cuts to the base model:

$$0 \leq \sum_{j'\in 1..n_{i'}} B_{i(j+1)i'j'} - \sum_{j'\in 1..n_{i'}} B_{iji'j'} \leq \left(\left\lfloor\frac{h_i - 2\times\underline{obj}}{h_{i'}}\right\rfloor + 1\right)$$
$$\forall (i,j),\ \&i' \mid i, i' \in F, j \in \{1 \dots n_i - 1\}, i < i'. \tag{9}$$

Similar to Proposition 3.2, we can bound the number of arrivals from all farms that can occur between two successive loads from a given farm. The proof is analogous to that of Proposition 3.2 and is omitted.

Proposition 3.3. *Given a feasible solution value $\underline{obj}$ and for two successive loads arriving to the mill from farm i, the number of maximum arrivals originating from any farm $i' \neq i$ is bounded by $\left(\left\lfloor\frac{h_i - 2\times\underline{obj}}{\underline{obj}}\right\rfloor + 1\right)$.*

As was the case with Constraints (9), to implement Proposition 3.3, we need to count the arrivals that occur between two successive loads from the same farm. We make use of the following sums:

$$\sum_{i'>i} \sum_{j'\in 1..n_{i'}} B_{i(j+1)i'j'}, \tag{10}$$

$$\sum_{i'>i} \sum_{j'\in 1..n_{i'}} B_{iji'j'}, \tag{11}$$

$$\sum_{i'<i} \sum_{j'\in 1..n_{i'}} \left(1 - B_{i'j'i(j+1)}\right), \text{ and} \tag{12}$$

$$\sum_{i'<i} \sum_{j'\in 1..n_{i'}} \left(1 - B_{i'j'ij}\right). \tag{13}$$

The sum (11) counts the total number of arrivals prior to $x_{i,j+1}$, and the sum (12) counts the total number of arrivals prior to $x_{i,j}$ from the farms with index i' greater than i. The sum (13) counts the total number of arrivals prior to $x_{i,j+1}$, and the sum (14) counts the total number of arrivals prior to $x_{i,j}$ from the farms with index i' less than i.

Thus, as a result of Proposition 3.3, we add the following optimality cuts when a feasible solution is available:

$$0 \leq \sum_{i'>i} \sum_{j' \in 1..n_{i'}} B_{i(j+1)i'j'} - \sum_{i'>i} \sum_{j' \in 1..n_{i'}} B_{iji'j'} + \sum_{i'<i} \sum_{j' \in 1..n_{i'}} \left(1 - B_{i'j'i(j+1)}\right) - \sum_{i'<i} \sum_{j' \in 1..n_{i'}} \left(1 - B_{i'j'ij}\right)$$
$$\leq \left(\left\lfloor \frac{h_i - 2 \times \underline{\underline{obj}}}{\underline{obj}} \right\rfloor + 1 \right) \forall (i,j), \mid i, i' \in F, j \in \{1 \ldots n_i - 1\}. \tag{14}$$

4. Instances and Solution Approach

To compare the approach presented in this paper to those in the literature, we use the 11 instances based on conditions in Louisiana. The instances were introduced in Lamsal, Jones, and Thomas (2013) and were designed to be as realistic as possible. The instances are based on data from National Agricultural Statistics Service (2013) and American Sugar Cane League (2013) that provide zip-code level addresses for 456 farms and exact addresses of the 11 mills. In total, the 11 instances represent 456 farms in 85 zip codes with a daily capacity of 4044 loads. The zip codes with sugarcane farms and the location of the mills are shown in Figure 1. Each star represents the location of a mill, and each dot represents the centroid of the zip code that has at least one sugarcane farm. Using this data as well as additional data from Barker (2007) and Salassi and Barker (2008), each farm is assigned a daily harvest volume and either one or two harvesters. We determine the harvest time per load from the number of harvesters. Farms are assigned to mills by solving a capacitated assignment problem for which the objective is to minimize the sum of the distances between the farms and the mill that serves the respective farms. We assume that each mill serves approximately the same number of loads. Table 1 summarizes the 11 mill areas. For the purposes of this study, it is assumed that each mill has the capacity to unload one truck at a time and that the unloading time takes two minutes. This choice facilitates comparison with Lamsal, Jones, and Thomas (2013), but also reflects approximately the time needed to feasibly service all loads during daylight hours.

Instance	# of farms	# of loads
1	55	370
2	66	367
3	29	369
4	39	365
5	23	365
6	69	370
7	28	370
8	53	370
9	27	365
10	26	365
11	41	368

Table 1. Distribution of farms and total loads

The integer programming model presented in Section 3 is solved using the branch-and-bound algorithm of GUROBI OPTIMIZER 5.6 using the Python interface. The experiments are performed on a 3.40 GHz Intel Core i7-3770 CPU running the Ubuntu 12.04 operating system. We tested the base model as well as various combinations of Constraints (7), (8), (9), and (14) with it. Using the base model alone or without all of the valid inequalities and optimality cuts, we are able to find feasible solutions, but we were unable to solve the problem to optimality given 1800 seconds

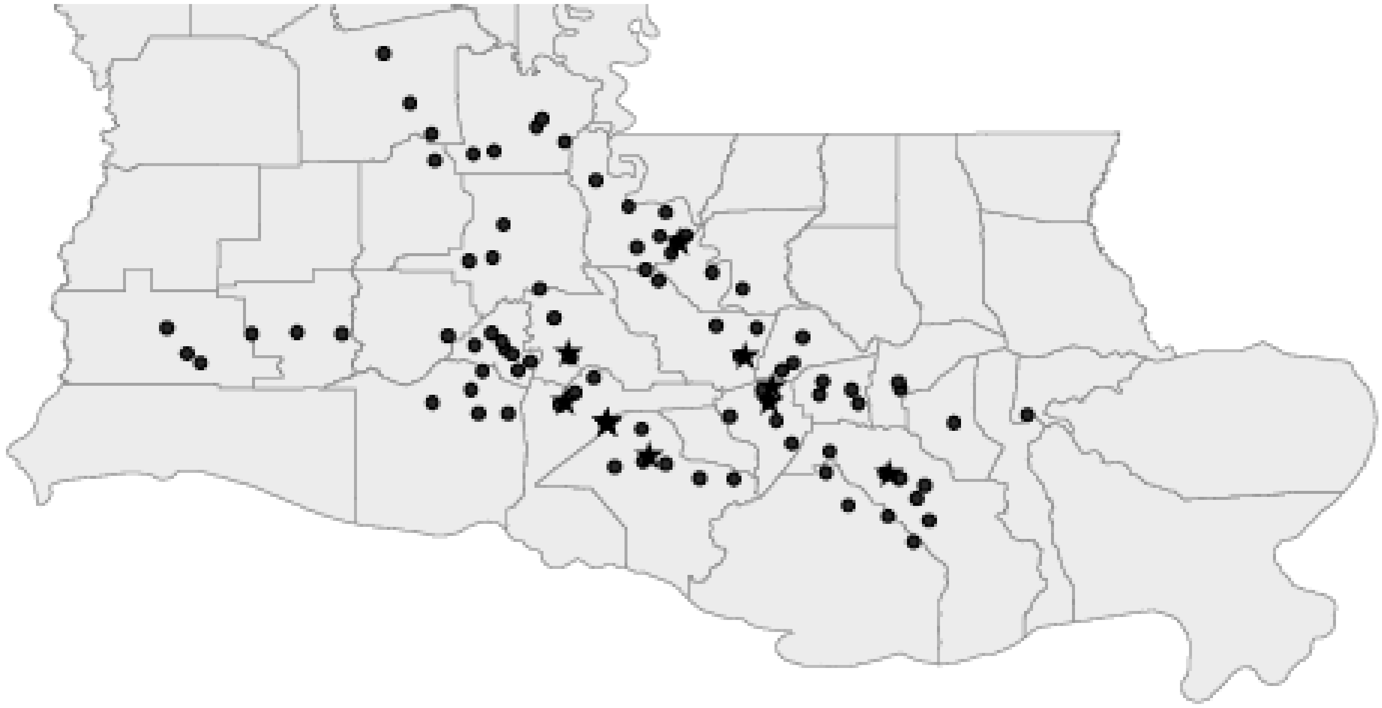

Figure 1. Mills and Centroids of Zip Codes containing Sugarcane Farms in southern Louisiana

of runtime. Whereas, with the valid inequalities, optimality cuts, and an initial feasible solution, each of the instances discussed previously can be solved to optimality in about two hours.

For this problem, the initial feasible solution is necessary not only for the computational advantages it provides, but also to instantiate Constraints (9) and (14). We use the model and algorithm described in Lamsal, Jones, and Thomas (2013) with hourly time blocks to get an initial feasible solution to the model presented in this paper. Because solutions to the model in Lamsal, Jones, and Thomas (2013) can have two arrivals that occur at the same time, we iteratively perturb the start times of the farms whose loads have the same arrival times until we have a solution in which no two loads have the same arrival times. As there are infinite real numbers, we are guaranteed to find a non-zero solution. For example, for two farms A and B that have loads arriving to the mill at the same time, we greedily decrease the start time of farm A by $\epsilon = [0.05, 0.25]$ and increase the start time of farm B by ϵ. Constraints (9) and (14) are instantiated using the objective of the initial feasible solution as the lower bound.

Each time the branch-and-bound algorithm finds a new incumbent solution, Constraints (9) and (14) are updated and added as new optimality cuts. The algorithm stops when the optimality condition is satisfied.

Given a solution to the math program, we can compute the number of trucks needed to transport the loads to the mill by their prescribed arrival times using the truck assignment algorithm presented Lamsal, Jones, and Thomas (2013). This is the second phase of our solution approach. Lamsal, Jones, and Thomas (2013) prove that, for an arbitrary set of arrival times, the algorithm finds the optimal number of trucks needed to deliver the loads at their respective arrival times. For completeness, we present the algorithm in the Appendix A. We note that the algorithm assumes that the trucks must wait in queue to be unloaded at the mill. However, the algorithm can easily be modified to include the case that the trucks are dropping fully loaded trailers and picking up empty trailers on return to the mill. The truck assignment algorithm is coded in Python and runs instantaneously on the previously described hardware.

5. Computational Results

This section presents a series of computational results. With these results, we seek to determine whether or not the continuous model has an advantage over the discrete model in Lamsal, Jones, and Thomas (2013). This question is motivated by the observation in Lamsal, Jones, and Thomas (2013) that, as time blocks become smaller, the model produces solutions using fewer trucks and spreading load arrivals more uniformly throughout the day. Lamsal, Jones, and Thomas (2013) also shows that the complexity of the problem increases when the size of the time blocks decreases to the extent that making the size of the time blocks smaller than 10 minutes (thus resulting in large number of blocks) produced unsolvable problems. For practical purposes, the continuous model proposed in this paper is equivalent to having infinitesimal time blocks.

We first compare our solutions with the solutions in Lamsal, Jones, and Thomas (2013) for the number of trucks needed to pick up all the loads at their ready times. A side-by-side comparison of number of trucks needed is presented in Table 2. The approach presented in this paper reduces the number of trucks in all but one instance (Instance 3), in which case the number of trucks are equal. On average, the number of trucks is reduced by 7%.

Instance	# of trucks needed (old)	# of trucks needed (new)
1	32	31
2	36	33
3	31	31
4	34	29
5	35	32
6	48	42
7	30	30
8	35	34
9	38	34
10	28	26
11	33	30

Table 2. Comparison of number of trucks needed to haul the cane in ready times

To understand why the approach presented in this paper reduces the number of trucks, we compare the two solution methods with respect to truck utilization. Figure 2 compares the time spent by each truck arriving to the mill in our solution to the solution in Lamsal, Jones, and Thomas (2013) for the first Louisiana instance. In this comparison, we make the truck assignments for both our and Lamsal et al.'s (2013b) solution by assuming a FIFO queue with unloading time of two minutes per load at the mill. It is noticeable that the waiting time for each load is shorter in our solution in-spite of having the same unloading time. The regular pattern for the hourly block solution is because of the staggering of the loads at the hour ends. Enforcing artificial hourly or half-hourly blocks and fitting the predefined numbers of arrivals in each of these blocks coupled with having to pick up the loads at their ready times adds unnecessary trucks just to make the loads arrive within the time blocks. Thus, the overall utilization of the fleet is reduced. Eliminating discrete time blocks, we allow of the flexibility of not having to stagger the arrivals around the block ends to meet the block's quota.

Further evidence of the value of the method presented in this paper can be seen by comparing variation in truck hours. We define truck hours for a truck as the time between when the last load hauled by the truck is unloaded at the mill and the time when the truck is dispatched from the mill to pick up the truck's first load. A better solution would reduce the variability in truck hours across all the trucks. That is, the trucks would all work about the same number of hours. One of the weaknesses of the solutions in Lamsal, Jones, and Thomas (2013) is that a significant number of trucks serve a single load. Thus, variability in truck hours is high in those solutions. Such reduction

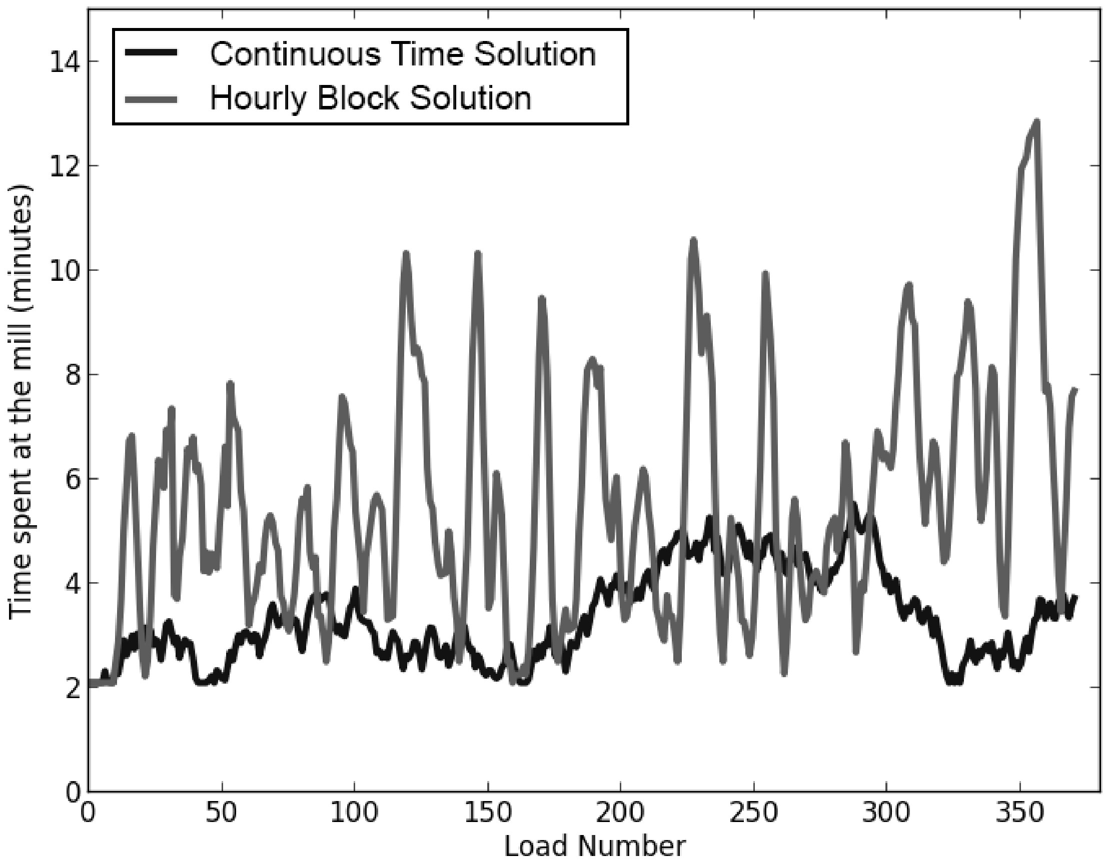

Figure 2. Time spent at the mill yard for individual load

in variability in truck hours should be desirable because it would be useful to equitably divide work among drivers.

In Table 3, "old STDEV" refers to the standard deviation of the truck hours, and "old Max - Min" refers to the difference between the maximum and minimum truck hours for each solution using the best solutions from (Lamsal, Jones, and Thomas 2013). Similarly, "new STDEV" and "new Max - Min" refer to the standard deviation and the difference between the maximum and minimum of the truck hours for our solutions. The approach presented in this paper reduces this variability by an average of 19% across the 11 mill areas. The difference between the maximum and minimum truck hours is also reduced by about 11.66 %.

Figure 3 plots the cumulative arrivals with three different solutions for the first instance. The line labelled as "Earliest Start for all farms," represents the solution that simulates the current practice (as suggested by Salassi and Barker (2008)) in which all farms start harvesting at the beginning of the day. The line labelled as "Hourly Block Solution" represents the time block solution obtained using the solution method described in Lamsal, Jones, and Thomas (2013) (Hourly blocks and 29 loads per hour limit). The line labelled as "Continuous TimeSolution" represents the solution from our proposed solution method. Our best estimate for the number of trucks needed to pick up all the loads for the first solution is 62 trucks. Similarly, we need 32 and 31 trucks, respectively for the second and third solution.

In "Earliest Start for all farms" solution, most loads arrive at the mill within the 500 minutes. This causes congestion at the mill increasing the turn around times for the trucks, thus increasing the number of trucks required to haul all the cane. In "Hourly Block Solution," the hourly truck

Instance	old STDEV	old Max - Min	new STDEV	new Max - Min
1	131.35	601	95.79	478
2	159.28	689	130.38	635
3	140.76	579	90.90	460
4	136.35	512	125.95	460
5	125.85	438	116.29	374
6	132.27	588	96.83	557
7	114.76	418	96.65	378
8	135.59	543	110.02	508
9	170.42	633	137.14	633
10	147.01	570	139.21	412
11	134.17	553	116.13	522

Table 3. Comparison of standard deviations and differences in working time

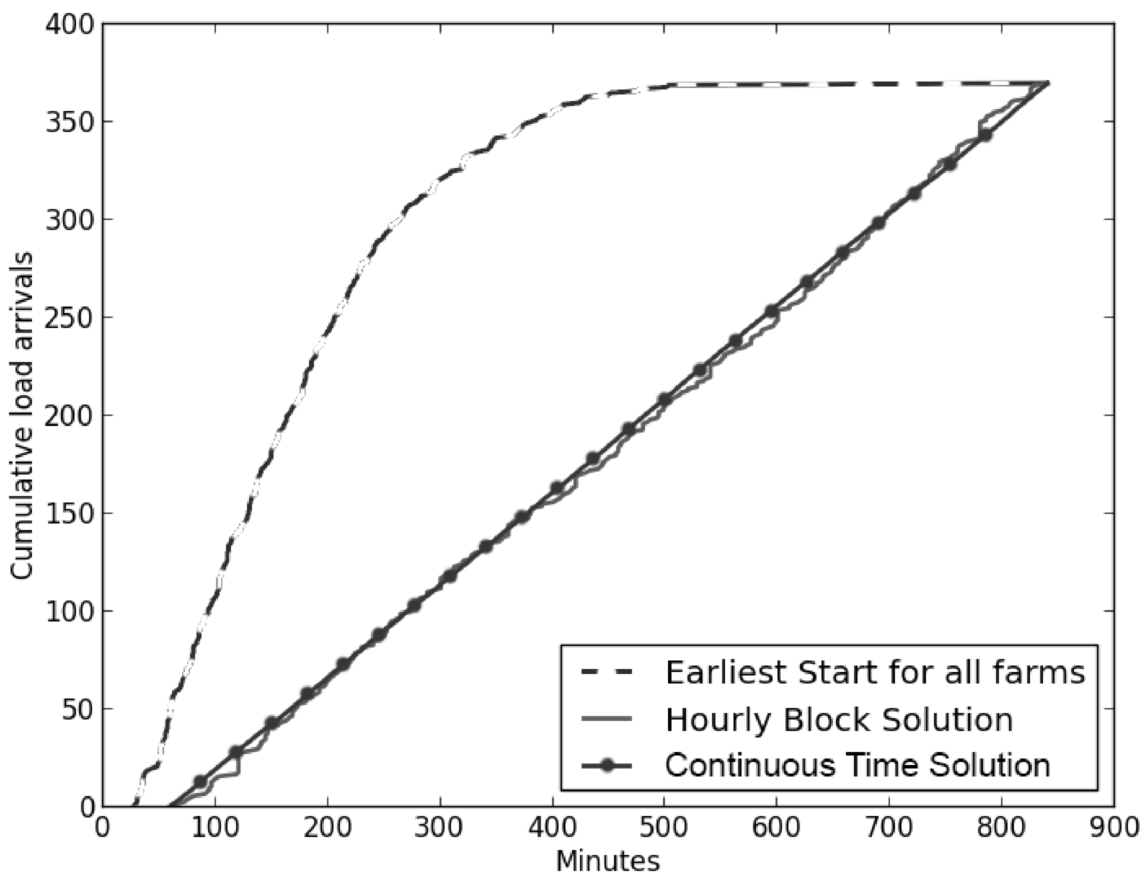

Figure 3. Cumulative arrivals at the Mill throughout the daylight hours

arrival rate is constant but within the hour, truck arrivals are not spread out. So, there are times, when the unloading resource at the mill is idle and there are also times when there is congestion as loads arrive simultaneously. In the "Continuous TimeSolution," the trucks arrive at a nearly constant rate, reducing the chances of the unloading resource at the mill being idle or the chances of congestion.

6. Conclusions

Optimizing operations in a sugar mill area is a difficult task involving several stake holders with competing interests. Previous literature in the area uses a discrete time approach that results in problems becoming computationally intractable as the time discretization becomes finer. This paper uses an objective function, maximizing the minimum difference between two consecutive arrivals at the mill, which allows the problem to be solved in continuous time; thereby obviating difficulties encountered using previous approaches. Our results show that this new approach provides solutions that not only reduce the number of trucks needed to conduct the harvest, but that also reduce variation of truck utilization. Reducing such variation is important for a variety of efficiency and operational reasons, but also because it spreads the workload more evenly amongst truck drivers, thereby increasing perceived fairness and equity. Additionally, our results show that these advantages can be obtained with only minimal coordination between the mills and farms. Notably, the farms must allow the mill to set the time of day at which the sugar cane harvest starts. Because the farms are independently owned, such minimal coordination requirements are important if the solution is to be workable in a practical setting.

There are three areas of future work. First, this paper considers the coordination of harvests through the practical mechanism of scheduling the start of harvests. With a longer time horizon in mind, it might be worth considering alternatives. For one, in Louisiana at least, each farm is currently harvesting every day as a means of providing equity to farmers. In particular, this framework means that no farmer has a chance to harvest the sugarcane when it is more ripe, and thus higher in sugar content and more valuable, than another. Future work could consider payment mechanisms that offer equity to the farmers while creating opportunities to reduce harvest costs, particularly through reduced transportation and equipment costs.

In addition, while this paper focuses on harvests in Louisiana specifically, we believe that there are additional areas in which our work is useful. First, there are many commonalities between the Australian case described in Higgins and Laredo (2006) and the work in this paper. Further, our work can be extended to harvest logistics for any agricultural system in which there are many producers and no on-site storage. Sugar beets and many vegetable crops are examples of such agricultural systems.

A third opportunity for future work is to explore methods for managing harvest logistics in real time. Although the model in this paper could be used to determine the start time of the daily harvest, additional work is needed before it could handle unknown events that might arise during a day‘s operations as they occur.

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Appendix A. Truck Assignment Algorithm

A solution of an instance of the integer program presented in Section 3 returns, for each load from each field, a prescribed time at which the load is to arrive at the mill. For any such set of prescribed arrival times, Algorithm 1 finds the optimal number of vehicles needed to transport loads from the farms to the mill so that the loads arrive at these prescribed times. The algorithm operates on truck dispatch and availability times. For a given load, the dispatch time is the latest time at which a vehicle needs to depart the mill so that it can travel to the appropriate farm and back to mill so that the load arrives at the mill at its prescribed arrival time. The availability times are the times at which a vehicle becomes available again for dispatch after leaving the mill to pick a load, returning to the mill, and being unloaded. Given the set of dispatch times and because the problem is deterministic, the availability times are straightforward to compute, even when vehicles need to wait in a line of trucks to be unloaded.

Algorithm 1 Algorithm for Finding the minimum number of vehicles Needed to Meet the Ready Times of the Loads

Input:

Conjoined and then sorted (in an ascending order), a list $\mathcal{L}$ of all dispatch times required to meet the arrival times and all the associated availability times of trucks for all loads. Accordingly, the k^{th} member of this list will be either a dispatch time or an availability time. The type is identified by a mapping $type(k)$.

Output: Minimum number of vehicles needed to meet the given arrival times of the loads.

Initialization:

```
  Trucks Used = 0
  Trucks Needed = 0
  k = 1
for k = 1 to |L| do
    if type(k) = dispatch then
        Trucks Used ← Trucks Used + 1
        Trucks Needed← max{Trucks Used, Trucks Needed}
    else
        Trucks Used ← Trucks Used - 1
    end if
    k ← k + 1
end for
```