# Harvest Logistics in Agricultural Systems with Multiple,

# Independent Producers and No On-Farm Storage

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#### Abstract

The best way to organize the logistics of harvesting agricultural crops requires considering not only the fact that agricultural commodities in general are highly perishable, but also the fact that the organizational structure of the agricultural system in question can vary from crop to crop and from region to region within a single crop. This paper develops a model for planning the movement of the crop from farm to processing plant for crops satisfying two conditions: (1) there are multiple, independent producers (farmers), and (2) no significant on-farm storage exists. We will also briefly describe three different but economically significant agricultural systems in the United States: sugarcane in Louisiana, sugar beets in the northern areas of the United States eg. South Datoka, Minnesota, Colorado, and vegetable harvesting for human consumption, and will argue that these systems fit the two conditions of our model. We will also briefly explain why several other significant agricultural systems do not fit these two conditions and hence require alternative modeling techniques. Finally, we demonstrate that the model developed in this paper is computationally tractable by introducing new datasets based upon the sugarcane industry in Louisiana. This choice was driven, not only by the fact that the datasets can be constructed entirely using publically available information on the sugarcane infrastructure in Louisiana,

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but by the fact that this particular organizational structure also appears in both the sugar beet and vegetable processing industries.

# 1 Introduction

This paper considers a supply chain problem in the agricultural industry that appears in a number of different settings. The problem is that of organizing the logistics of transporting the crop from producers' fields to a single facility at which the crop may be processed for further supply chain distribution or, in the case of sugar beets, stored for future processing at a different location. We consider a class of problems satisfying two conditions: (1) there are multiple, independent producers; and (2) on-farm storage of the harvested crop is impractical or impossible.

The first condition rules out vertically integrated corporations in which the processing facility and the farms are owned by the same entity. This precludes the possibility of allowing a coordinator at the centrally located processing facility to continuously monitor and vary harvesting rates at individual farms as decision variables. While varying harvest rates at individual farms might seem a tad bit far-fetched, it actually arises in the Brazilian sugarcane industry in which vertical integration is the norm and a central controller varies harvesting rates at the fields (fronts) being harvested as conditions warrant. The second condition, that of no on-farm storage, rules out a number of important crops such as corn for which on-farm storage, while not universal, is still quite common with many farms having on-site drying cribs as well as storage cribs in which corn may be stored for years prior to sale or subsequent use. The second condition also implies that the harvested crop is transported from the farm as it is harvested.

The situation we address in this paper is one of planning the logistics for a single 24 hour period. During this day, the crop is harvested at a set of harvest locations (farms). The crop is moved from the harvest locations to the single facility (for storage or processing) in trucks which then return to the farms to pick up another load. Each harvest location has a pre-specified number of loads that are available to be harvested on the day in question. The locations of the fields being harvested are known as is the location of the single facility to which the crop is being removed. The travel times from each harvest location to and from the single facility are known as are both the time for loading a transport vehicle at the farm and the time for unloading the transport vehicle at the facility. We assume that, although the harvest rates at each harvest location are constant, the start times of the harvest at each harvest location are variables that we can set.

The model we propose in this paper divides the day into blocks of time which can be of arbitrary length. For each time block, there is a pre-defined unloading target at the single facility. Since the time blocks can be of different length, so can the unloading targets be different from one time block to the next. Actual deliveries of the crop to the facility are, of course, constrained by the availability of resources within the harvesting and transportation system associated with the facility. We take as our objective to minimize the cumulative deviation of actual deliveries to the facility from the desired unloading targets throughout the day.

Note that the unloading target during a time block may be determined by the actual capacity of the facility to process the crop. The objective makes sense in that it spreads the arrivals of crop transport vehicles at the facility to meet the ability of the facility to process those loads. In the event that the processing capacity at the facility remains constant over the day, the objective will attempt to minimize the variance of the arrival stream of transport vehicles at the facility. In the parlance of queueing systems, we will have reduced as far as is possible the variance of the customer arrival stream. Consider that the time spent in queue in a single server queueing system are given by the equation

$$p \times \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} \times \frac{CV_a^2 + CV_p^2}{2},\tag{1}$$

where p is the average time required to unload a single crop transport vehicle at the single facility,  $\frac{\lambda}{\mu}$  denotes the utilization of the single facility, and values  $CV_a$  and  $CV_p$  denote the coefficients of variation in the arrival stream and in the processing stream, respectively.

To see why minimizing the variation in the arrival stream makes sense when considering the problem of managing the crop harvest and transport system, note that other parameters in Equation (1) are properties of the facility and hence lie outside the realm of the crop transport system. Our stated objective is aimed at reducing the coefficient of variability of the arrival stream and, ceteris paribus, the average time that vehicles spend queueing at the facility. Reducing time in queue reduces congestion at the facility, thereby making more effective use of available crop transport vehicles. In the case of the sugar industry discussed in Section 2, it also will shorten the time between harvest and pre-cooling (for sugar beets) or the time between harvest and crushing (for sugarcane) thereby reducing sugar losses. In the case of the vegetable industry, reducing the harvest to process time is also of considerable importance since otherwise the quality of the resulting produce is significantly lessened.

This paper makes several contributions to the literature. First, the paper introduces a tractable two-phase solution approach that overcomes computational challenges seen in analogous problems in the literature. Second, the paper introduces a novel use of a technique from the piecewise linearization of functions to linearize discrete time blocks. The paper also introduces a provably optimal algorithm for determining the number of trucks needed to serve the harvested loads. Finally, our computational results demonstrate not only the computational effectiveness of our approach, but also that spreading the arrival of vehicles as evenly as possible at the mill reduces the number of trucks needed to serve the harvest.

In what follows, Section 2 describes three areas of agriculture in which our model and solution approach is relevant. Section 3 presents a literature review. Section 4 describes our solution approach and presents a formalization of the model that we have presented verbally in the introduction. Section 5 introduces a set of test instances and presents the results of experiments that demonstrate the effectiveness of our approach. The set of test problems are derived from publicly available data on the geographical locations of each of Louisiana's 456 sugarcane farms and 11 sugarcane mills as well as their production and processing rates. These instances allow us to demonstrate the ability of our method to solve realistically sized problems. Section 6 concludes the paper.

## 2 Application Areas

In this section of the paper, we describe three examples of agricultural harvesting and transportation systems that fall into the class of problems covered by the model and solution approach presented in this paper. The examples are: a) sugar beet crops, b) sugar cane crops, and c) vegetable crops. We restrict ourselves in this paper to agricultural systems as they are organized and practiced within the United States. Significant variations exist between countries even for the same crop due to differing factors including ownership structures, climactic conditions, operational scales, and technological infrastructure. Doubtless, there are additional examples in other countries as well as in the United States that satisfy the conditions of the modeling work in this paper, but our purpose here is not to develop a comprehensive catalog of agricultural supply chain problems that fit into this framework, but rather to show that this framework has nontrivial applications. Three significant examples within United States agriculture seem sufficient for this purpose.

#### 2.1 Sugar beet crops

Sugar beets are an economically significant crop in the United States. In 2014, 1,147,000 acres were harvested, yielding approximately 4.88 million tons of sugar (United States Department of Agriculture Economic Research Service, 2015). Estimating the economic value of this crop by the average world raw sugar price of 16.34 cents per pound (United States Department of Agriculture Economic Research Service, 2015), which is less than the prevailing US price due to governmental price supports during the year in question, we get an estimated economic value for the crop of approximately \$1.595 billion. The calculated value would, of course, be higher had we used the actual prevailing price in the U.S., but doing so would overstate the true economic value of the crop due to the government price supports.

In the United States, sugar beets are grown primarily in the northern plains states because the weather conditions are favorable. It is typical for a sugar beet farmers to belong to a coop that owns one or more factories that process sugar beets into sugar and associated by-products. The Southern Minnesota Sugar Beet Cooperative, for example, has 500 growers and operates 12 receiving stations as well as one factory for processing the sugar beets (Souther Minnesota Beet Sugar Cooperative, 2015). The Western Sugar Cooperative, as another example, has over 1000 growers and operates 43 receiving stations, 7 storage locations, and 5 processing plants (Western Sugar Cooperative, 2015).

Sugar beets reach their peak sugar content in early October and begin to lose sugar thereafter until, if harvesting is delayed until January, they may have lost up to 85% of their initial sugar content. Furthermore, sugar beets deteriorate immediately after harvest, and the sugar loss varies directly with the temperature at which the sugar beets are stored (Investment Centre Division of the Food and Agriculture of the United Nations, 2009).

These facts seem to drive much of the protocol followed in the United States for harvesting and processing sugar beets. In the United States, the sugar beet harvest begins with a small pre-pile harvest in September that allows growers to open up roadways through fields in preparation for the main harvest which begins in October. The pre-pile harvest is quite limited due to the warm weather conditions and is delivered directly to the processing facility, by-passing the receiving stations entirely.

In October, the main harvest begins and growers deliver their sugar beets to a designated receiving station where they are off-loaded onto sugar beet pilers that pile the sugar beets into large storage piles that can be up to 20 feet tall and 1500 feet long. In most years, the cold weather in northern plains states offers an advantage as it permits outdoor storage of sugar beets in below-freezing weather conditions thereby minimizing sugar losses. To further minimize losses, many organizations use fans to circulate cold outside air through ventilation channels that are constructed within sugar beet piles to pre-cool and then deep-freeze the sugar beets. To extend the sugar beet storage period, some organizations also use large enclosed sheds at their factories or other storage locations in which fan ventilation is used to deep freeze stored sugar beets.

The harvest is normally concluded by the end of October, but factories continue to operate for up to 6 months to process the stored sugar beets. The cooler weather in northern plains states not only allows the growers to complete their harvest during October when sugar content is higher, it also permits longterm sugar beet storage which allows a smaller factory to process the year's crop over a longer period than would be the case if long term storage was rendered impossible by the warmer weather encountered in other climates.

It is apparent that the system for harvesting and transporting sugar beets satisfies the two conditions for membership in the class of problems we address in this paper. First, on-farm storage of the harvested crop is impractical due to the capital equipment requirements necessary to construct and operate piles in which sugar beets can be pre-cooled and then deep-frozen for long term storage. Second, from the two examples cited earlier, it is obvious that each sugar beet receiving station and processing facility is supported by a large number of independent growers.

### 2.2 Sugar cane crops

According to the USDA, only four of the United States currently produce sugarcane, with Louisiana and Florida being by far and away the largest producing states (United States Department of Agriculture Economic Research Service, 2015). In 2014, for example, the production (in thousands of short tons) was 1759 for Florida, 168 for Hawaii, 1583 for Louisiana, and 145 for Texas. Hawaii and Texas have vertically integrated sugarcane industries with one firm in each state. Florida has 1 sugar mill that is cooperatively owned by sugarcane growers, but the remaining 5 mills are privately owned and vertically integrated. In Louisiana, in contrast, there are 11 sugar mills supported by approximately 475 sugarcane growers. According to the USDA, approximately 45% of US sugar production is from sugar cane while the remaining 55% is almost entirely from sugar beets.

Sugarcane is harvested on farms by harvesters that cut the sugarcane stalks into shorter billets and then load them into field transport units that are then pulled to a central loading facility on the farm. The field transport units are then emptied into trucks that transport the cut sugarcane stalks directly to the mill where they are unloaded and crushed to extract the sugarcane juice.

Sugarcane shares some similarities with sugar beets in that, like sugar beets, the sugar content of sugarcane begins to decline immediately upon harvest due largely to evaporative losses. Since sugar cane is grown only in warm climates, the outdoor piling, cooling, and deep freezing that serves to arrest sugar losses in the sugar beets grown in northern plains states is impossible for sugarcane. As a result there are no analogs to receiving stations in the sugarcane industry, so the cut cane is transported directly to the mill. To reduce sugar losses from cut sugar cane, the cut to crush time must be kept to a minimum, so logistical solutions, to be of practical importance, must address this need. That means that practical solutions must reduce the need for safety stock inventory of cut sugarcane at the mill, which can best be done by reducing variation in the system, thereby reducing congestion and consequent waiting time at the mill.

The system for harvesting and transporting sugar cane always satisfies the first condition for membership in the class of problems we address in this paper because on-farm storage of the harvested sugarcane crop is impractical and is never done regardless of location. Second, for both the sugar cane co-op in Florida and the American Sugar Cane League in Louisiana, it is apparent that each sugar mill is supported by a large number of independently owned growers. Thus, a significant portion of the US Sugarcane industry satisfies the two conditions necessary for membership in the class of problems we address in this paper.

### 2.3 Vegetable crops

The USDA estimates that, during the years 2000 - 2007, U.S. farm cash receipts from the sale of vegetables averaged \$17.4 billion (United States Department of Agriculture Economic Research Service, 2015). The vegetable industry can be divided into two major end uses, fresh market and processing with processing sub-divided into canning, freezing, and dehydrating. About half of the vegetable production in the U.S. goes into each of the two end uses. In the processing segment of the vegetable industry, contractual agreements between growers and processors are the norm, but contracting in the fresh market segment is less common.

Generally, vegetables are grown either for fresh market use or processing, with little substitution taking place. This is because varieties grown for processing are usually better adapted to mechanical harvesting and have characteristics desirable for processing rather than for fresh-market sale. The USDA notes, for example, that tomatoes, with the exception of plum tomatoes, grown for processing tend to be smaller and have different internal characteristics than most fresh varieties. Fresh vegetables are grown throughout the United States, and it is worth noting that climate causes most domestic fruit and vegetable production to be seasonal, with the largest harvests occurring during the summer and fall.

The USDA 2012 Agricultural Census lists 72,045 farms producing vegetables within the U.S. with approximately 13,000 of them producing for the processing segment and approximately 66,000 producing for the fresh market segment - note that some produce for both segments and hence are double counted (National Agricultural Statistics Service, 2012). Of these 72,045 total farms, 57,194 (or approximately 79% of the total) are of less than 15 acres, so it is safe to say that the vegetable growing industry is comprised of numerous small and independent growers. Fresh vegetables in virtually all cases must be moved quickly after being harvested (whether mechanically or by hand) to the processing facility to avoid deterioration and spoilage. It is apparent that on-farm storage as is commonly observed with dried grains is not possible for the majority of vegetable crops. Thus, significant portions of the vegetable crop industry satisfy the two conditions necessary for membership in the class of problems we consider in this paper.

## 3 Literature Review

The operations research literature has considered a broad range of applications. Recent reviews can be found in Lowe and Preckel (2004) and Ahumada and Villalobos (2009). Harvest logistics in particular have been the subject of a large number of papers, but these papers have, for the most part, focused on the sugarcane industry. Because of different infrastructures and differing levels of vertical integration, however, the application of such papers is usually country and industry specific. Hansen et al. (2002), Lejars et al. (2008), Le Gal et al. (2009), and McDonald et al. (2008) explore supply chain issues in the South African sugarcane industry. Supsomboon and Yosnual (2004) and Prichanont et al. (2005) study uncertainty arising in the Thai sugarcane industry. Diaz and Perez (2000), López-Milán et al. (2006), López-Milán and Plà-Aragonés (2013), and López-Milán and Plà-Aragonés (2015) explore the multi-model sugarcane logistics in Cuba. Hildebrand (2002), Higgins and Postma (2004), Higgins and Davies (2005), Higgins and Laredo (2006), and Higgins (2006) explore sugarcane supply logistics in Australia. Salassi and Barker (2008) and Salassi et al. (2009) study the logistics of harvesting sugarcane in Louisiana, in the United States, and conjecture that coordinating harvest start times in the fields might evenly spread out the delivery of sugarcane to the mill. We also coordinate start times at the farms in this paper. Finally, Lamsal et al. (2015) study the coordination of harvest logistics in Brazil and provide a detailed review of harvest logistics around the world.

Papers outside of the sugarcane do not directly discuss the coordination of daily harvests and transportation as is the case in this paper. However, there has been increasing interest in the agro-supply chain, particularly as a result of the interest in the consumption of locally grown food. Coelho and Laporte (2014) discuss the history of distribution of perishable goods, such as vegetables, to retailers and provide a combined delivery and inventory policy for such goods. Osvald and Stirn (2008) and Chen et al. (2009) focus on the last mile problem of supply markets with fresh produce. Nguyen et al. (2014) considers the consolidation of supplier shipments to wholesalers in the fresh-cut flower industry. Etemadnia et al. (2013) and Etemadnia et al. (2015) consider the location of hubs that would facilitate better supply of regionally sourced produce for local markets. Bosona (2011) consider hub location as well as "milk run" routes to reduce the costs of bringing the produce of small producers to local markets. Filcek and Józefczyk (2012) considers the higher level problem of allocating farm supply to processing plants and the assignment of processed supply to retailers, with an application to sugar beets.

The work in this paper is most closely related to that of Higgins (2006) who studies the problem of truck congestion at an Australian sugar mill. The objective in Higgins (2006) is to minimize the sum of the mill's idle time and the trucks' queue time. The main difference with the problem studied in this paper is that, in the Australian case, the cane filled trailers can wait for the trucks returning from the mill at the fields. We also note that Higgins (2006)explicitly models the queue at the mill. Unfortunately, directly modeling the queue adversely impacts the tractability of the model. Consequently, Higgins (2006) applies both a tabu search and a variable neighborhood search to solve the problem. Higgins work provided a substantial advancement over the manual methods in place at the time he published. In this paper, we avoid the computational tractability issues alluded to above by dividing the problem into two phases. We do not explicitly model the queue in the first phase, thereby avoiding computational difficulties; but we do account for queueing impacts in the second phase.

### 4 Two-Phase Solution Approach

In this section, we present our two-phase solution approach. Our goal, as discussed in Section 1, is to minimize the variation in the arrival stream of trucks at the facility to which the crop is being transported. The model divides the day into blocks of time which can be of arbitrary length, but for each time block, there is a pre-defined unloading target at the single facility. Since the time blocks can be of different length, so can the unloading targets be different from one time block to the next. We take as our objective to minimize the cumulative deviation of actual deliveries to the facility from the desired unloading targets throughout the day.

Our decision variables are the start times for harvesting at the various harvest locations and the number of trucks used to transport the crop in from the field. Because solving the integrated problem is computationally intractable, we will adopt a two-phase solution approach.

In the first phase of our two-phase solution approach, we set the start time of the harvests in the fields. We model the harvesting rates, loading and unloading rates, and transportation times as known, deterministic parameters. Consequently, setting the start times determines the times at which loads of the crop arrive at the facility. In setting the start times, our objective of minimizing the cumulative deviation of actual deliveries to the facility from the desired unloading targets throughout the day can be viewed as one of matching the supply of crop at the facility to the demand for the crop at the facility. As discussed in the introduction, the objective is designed to minimize congestion at the mill and thus should have the effect of minimizing the number of trucks.

Given a solution to the first stage, the second stage determines the number of trucks required to serve each load at its ready time. For this purpose, we present an algorithm and prove that it determines the minimum number of trucks required to serve the given set of ready times with no waiting.

### 4.1 Model for Determining Harvest Start Times

The first step of the model is that we divide the daylight hours into blocks of time. A block can be of arbitrary length. For each of these blocks, the mill has a predefined unloading capacity, expressed as the number of loads that the mill can process in the block of time. The unloading capacity need not be the same for each block, reflecting potentially changing capacity throughout the day. We call this partition of the day P.

Suppose we have the partition P of the delivery window. As shown in Figure 1, a partition P, with n blocks can be specified by the set of n + 1 points. Specifically,  $\{a_k\}$ , for  $k = 0, 1, \dots n$ . We enforce  $a_0 < a_1 < a_2 \dots < a_n$ . The start time of the delivery window at the mill is  $a_0$  and  $a_n$  is the end time of the delivery window.

For each block k, the time between  $a_{k-1}$  and  $a_k$ , we have a predefined unloading capacity of  $N_k$  loads. We solve for a harvest schedule that matches load arrivals with the mill's capacity in each block and penalizes any deviation from the mill's desired unloading capacity in each block.

$$N_{0} N_{1} N_{2} N_{3} N_{n-1} N_{n}$$

Figure 1: Partition of the delivery window and the corresponding unloading capacity

Let  $x_{ij}$  be the arrival time at the mill of  $i^{th}$  farm's  $j^{th}$  load. Let  $y_i$  be the time when harvesting starts at farm i, and  $h_i$  be the time it takes to harvest a load at farm i. Let the travel time between farm i and the mill be  $t_i$ , and  $Q_i$ , the daily load quota of the farm i in F. The arrival time of a load to the mill depends on the harvest start time for the originating farm, the harvest rate at that farm, and the travel time between the farm and the mill. Mathematically,

$$x_{ij} = y_i + j \cdot h_i + t_i \qquad \forall i \in F, j \in \{1, \dots, Q_i\}$$

To distribute the loads among blocks, we need to count the number of load arrivals in each of these time blocks. A straightforward approach is to introduce binary variables  $b_{ij}^k$  for  $k = 1 \dots n$ , that indicate whether or not load j from field i arrives between  $a_{k-1}$  and  $a_k$ . With M as a large number and using the constraints

$$a_{k-1} \le x_{ij} + (1 - b_{ij}^k) \times M$$
  $\forall i \in F, j \in \{1, \dots, Q_i\}, k \in 1, \dots, n$  (2)

$$x_{ij} + (1 - b_{ij}^k) \times (-M) \le a_k \qquad i \in F, j \in \{1, \dots, Q_i\}, k \in 1, \dots, n$$
(3)

$$\sum_{k=1}^{n} b_{ij}^{k} = 1 \qquad \forall i \in F, j \in \{1, \dots, Q_i\},$$
(4)

we can determine in which block each arrival occurs.

While Constraints (2) to (4) may be straightforward, they lead to a formulation with a poor relaxation. Consequently, we propose an alternative that expresses the arrival time  $x_{ij}$  as a convex combination of the beginning and the end time of the section k in which the arrival time lies. Specifically,  $x_{ij}$  in section k can be expressed as:

$$\begin{split} x_{ij} &= \lambda_{ij}^{k-1} \times a_{k-1} + \lambda_{ij}^k \times a_k \\ \lambda_{ij}^{k-1} + \lambda_{ij}^k &= 1 \\ 0 &\leq \lambda_{ij}^{k-1} \leq 1 \\ 0 &\leq \lambda_{ij}^k \leq 1. \end{split}$$

More generally, we can write

$$x_{ij} = \sum_{k=1}^{n} \lambda_{ij}^k \times a_k, \qquad \sum_{k=1}^{n} \lambda_{ij}^k = 1, \qquad \lambda_{ij}^k \in \mathbb{R}^+ \qquad \forall i \in F, j \in \{1, \dots, Q_i\},$$

if for given *i* and *j*, we force at most two  $\lambda_{ij}^k$  variables among  $\lambda_{ij}^k$  variables,  $k = 0, 1, \dots n$ , to be positive. If  $\lambda_{ij}^k$  and  $\lambda_{ij}^l$  are positive, then k = l - 1 or k = l + 1. This situation can be modeled using binary variables  $b_{ij}^k$  for  $k = 1 \dots n$ , (where  $b_{ij}^k = 1$  if  $a_{k-1} \le x_{ij} \le a_k$  and  $b_{ij}^k = 0$ , otherwise), and the following constraints:

$$\begin{split} \lambda_{ij}^0 &\leq b_{ij}^1 \qquad \forall (i,j) \forall i \in F, j \in \{1, \dots, Q_i\} \\ \lambda_{ij}^k &\leq b_{ij}^{k-1} + b_{ij}^k \qquad \forall k \in 1, \dots n-1, i \in F, j \in \{1, \dots, Q_i\} \\ \lambda_{ij}^n &\leq b_{ij}^n \qquad \forall (i,j) \forall i \in F, j \in \{1, \dots, Q_i\} \\ \sum_{k=1}^n b_{ij}^k &= 1 \qquad \forall (i,j) \forall i \in F, j \in \{1, \dots, Q_i\}. \end{split}$$

To account for the situation when not all the required loads can be delivered or extra loads must be delivered during any section in the partition, we define slack and surplus variables,  $s_k^-$  and  $s_k^+$  for  $k = 1 \dots n$ . Then,

$$\sum_{i \in F} \sum_{j \in 1 \cdots n_i} b_{ij}^k + s_k^+ - s_k^- = N_k \qquad \forall k \in 1, \dots, n.$$

Now, we can express a complete mixed integer program (MIP) as:

 $\lambda_{ij}^k \in [0,1]$ 

$$\min \sum_{k \in 1 \cdots n} (s_k^+ + s_k^-)$$

$$x_{ij} = y_i + j \cdot h_i + t_i \qquad \forall i \in F, j \in \{1, \dots, Q_i\}$$
(5)

$$x_{ij} = \sum_{k=1} \lambda_{ij}^k \times a_k \qquad \forall i \in F, j \in \{1 \dots Q_i\}$$

$$\sum_{k=1}^n \lambda_{ij}^k = 1, \qquad \forall i \in F, j \in \{1 \dots Q_i\}$$

$$(6)$$

$$\forall i \in F, j \in \{1 \dots Q_i\}$$

$$\lambda_{ij}^{k} = 1, \qquad \forall i \in F, j \in \{1 \dots Q_i\}$$
(7)

$$\lambda_{ij}^0 \le b_{ij}^1 \qquad \forall i \in F, j \in \{1, \dots, Q_i\}$$

$$\lambda_{ii}^k \le b_{ii}^{k-1} + b_{ii}^k \qquad \forall i \in F, j \in \{1, \dots, Q_i\}, k \in 1, \dots, n-1$$
(9)

$$\lambda_{ij}^n \le b_{ij}^n \qquad \qquad \forall i \in F, j \in \{1, \dots, Q_i\}$$

$$(10)$$

$$\sum_{k=1}^{n} b_{ij}^{k} = 1 \qquad \forall i \in F, j \in \{1, \dots, Q_i\}$$
(11)

$$\sum_{i \in F} \sum_{j \in 1 \cdots n_i} b_{ij}^k + s_k^+ - s_k^- = N_k \qquad \forall k \in 1, \dots, n$$
(12)

$$b_{ij}^k \in \{0,1\}$$
  $\forall i \in F, j \in \{1,\dots,Q_i\}, k \in 1,\dots,n$  (13)

$$\forall i \in F, j \in \{1, \dots, Q_i\}, k \in 1, \dots, n$$
(14)

0.1

$$S_k^+, S_k^- \ge 0 \qquad \qquad \forall k \in 1, \dots, n$$

$$(15)$$

$$y_i \ge 0 \qquad \qquad \forall i \in F \tag{16}$$

$$x_{ij} \ge 0 \qquad \qquad \forall i \in F, j \in \{1, \dots, Q_i\}.$$

$$(17)$$

The objective function penalizes both positive and negative deviation from the mill's unloading capacity, but this does not always have to be the case. We can choose to just penalize the number of arrivals that are above the mill's capacity in the given period. Such a scenario might be appropriate when adding one extra arrival significantly affects the average turn around time for trucks. We can also choose to penalize just the deviation on the lower side. This might be appropriate when we want to achieve high utilization for the resources at the mill yard. We can also choose to penalize deviations in one direction more heavily than the deviation on the other side. We could also give more weight to the deviations in one period than the deviations in other periods.

Constraints (5) related the arrival times of the loads from each farm to the harvest start time at the

farm as well as the number loads harvested at the farm. Constraints (6) - (11) define each arrival time as a convex combination of the limits on the block into which arrival falls. Constraints (12) determine the level of violation in each time block. Finally, Constraints (13) - (17) appropriately constrain the variables.

For all subsequently presented instances and associated computational results, the solutions to the above MIP are obtained using GUROBI OPTIMIZER 5.6. The experiments were performed on a 2.3 GHz Intel Core i7 CPU running OS X 10.9. Communication with Gurobi was achieved through Gurobi's C++ interface.

### 4.2 Determining the Number of Trucks to Serve the Loads

In this section, we present an algorithm that determines the optimal number of trucks needed to serve a set of loads so that they arrive to the mill at their respective prescribed arrival times. In essence, the algorithm tracks the time required for each truck to travel to a farm to pick up a load, to travel back, and to unload. Once a truck has finished unloading, it becomes available to serve another load. If there is not a truck waiting at the mill when one must leave to pick up a load at the time it completes harvesting, the number of trucks is incremented. Algorithm 1 formally presents the procedure.

The algorithm assumes a set of dispatch times, or the times at which a truck must leave the mill to pick up a load and return it by its prescribed arrival time. Given a solution to an instance of the above MIP and the accompany values of the x variables, say  $\bar{x}$ , the dispatch time for each load can be calculated as  $\bar{x}_{ij} - 2 \times t_i$ . This computation reflects that the vehicle must travel a round trip from the mill to the farm to complete the delivery of a load at its arrival time. We note that, in the model presented in this paper, meeting the prescribed arrival time of a load also means that the load is picked up at the farm at the time at which harvesting is completed or  $y_i + j \times h_i$  for every i in F and j in  $1, \ldots, Q_i$ . Algorithm 1 also makes use of a list of vehicle availability times, which are the times at which vehicles become available after picking up a load at a front, returning to the mill, and unloading the load at the depot. For a load j from farm i, the corresponding availability time is

$$\bar{x}_{ij} - 2 \times t_i + \sigma_i,\tag{18}$$

where  $\sigma_i$  is the time required to unload at the mill a load from farm *i*. In other situations, it may be necessary to account for the fact that trucks must wait for others to be unloaded. In that case, the availability times must be computed algorithmically as not only does a vehicle incur the unloading time  $\sigma_i$ , but also time waiting for the vehicles in front of it at the mill to be unloaded. The computation of availability times given in Equation (18) can be viewed as a case in which the mill has unlimited unloading capacity and thus no queueing occurs.

Algorithm 1 Algorithm for Finding the Minimum Number of Vehicles Needed to Meet the Ready Times of the Loads

#### Input:

Conjoined and then sorted (in an ascending order), a list  $\mathcal{L}$  of all dispatch times required to meet the arrival times and all the associated availability times of trucks for all loads. Accordingly, the  $k^{th}$  member of this list will be either a dispatch time or an availability time. The type is identified by a mapping type(k).

**Output:** Minimum Number of Vehicles Needed to Meet the Given Arrival Times of the Loads.

### Initialization:

```
Trucks Used = 0
 Trucks Needed = 0
 k = 1
for k = 1 to |\mathcal{L}| do
     if type(k) = dispatch then
           Trucks Used \leftarrow Trucks Used + 1
           Trucks Needed \leftarrow \max\{\text{Trucks Used}, \text{Trucks Needed}\}
     else
           Trucks Used \leftarrow Trucks Used - 1
     end if
     k \leftarrow k+1
end for
```

**Proposition 1.** Algorithm 1 determines the number of vehicles necessary to serve each load at its respective ready time.

*Proof.* The correctness of the algorithm follows the if-statement in the for-loop. A truck is required only if the  $k^{th}$  element on the list is a dispatch. The number "Trucks Used" is incremented accordingly. The value of "Trucks Used" is decremented only if a truck becomes available. Thus, the value of "Trucks Needed" can increase only in cases where a truck is required for dispatch, but none is available. 

We also note that the algorithm has a complexity of O(|L|), resulting from the for loop and additions, subtractions, and comparisons required therein. However, sorting of the dispatch times is  $O(|L| \log |L|)$ .

Algorithm 1 is coded in Python and runs instantaneously on the previously described hardware.

# 5 Computational Experiments

In this section, we demonstrate that the model we have developed can actually solve problems of the sort that would actually be encountered in the agricultural industry. To this end, we develop a set of test problems based on publicly available data from the Louisiana sugarcane industry using actual locations of the nearly 500 sugarcane farms and 11 sugarcane mills in the state. We do not claim that these problems are in all details those encountered in the Louisiana sugarcane industry because we have had to approximate some details by making estimates of harvesting rates, travel times, and assignments of farms to mill that may not in all respects mirror reality. Nonetheless, the size and scope of the resulting test problems do accurately approximate those for at least one actual agricultural industry.

In solving the test problems, not only do we show that the computational approach presented in this paper can solve realistically sized problems, but we also show that, even though our only control variables at the individual farm level are the start times of harvests (unlike the situation in Brazil in which the vertically integrated industry and the technology employed permits continuously varying harvest rates at individual harvest locations), we are able to do a very good job of matching crop arrivals to crop processing needs at the mill. We also explore the limits of this modeling approach by showing how the approach works when we attempt to use a finer and finer discretization of the 24-hour time interval considered.

### 5.1 Datasets

In this subsection, we present the instances that we use to test the effectiveness of our approach. We make no claim that these test problems exactly replicate the Louisiana sugarcane harvesting system, but we do believe these test problems, since they are based upon actual location data of real farms and real processing facilities, represent the kinds of situations likely to be encountered in real planning situations.

Our instances are based on publicly available data for sugarcane farms and mills in Louisiana in the United Sates. There are 11 mills and approximately 475 farms in Louisiana. National Agricultural Statistics Service (2014) provide zip-code level addresses for 456 farms, and American Sugar Cane League (2013) provides exact addresses of the 11 mills. We also have the county level data on sizes of the farms that puts them into buckets of various sizes (National Agricultural Statistics Service, 2014). First, we calculate the distances between the farms and the mills. Then, we randomly assign the sizes for the individual farms according to the distribution of farm sizes in the respective counties. We then assume that farms that harvest more than 750 acres of cane a year have two combine harvesters and the ones that harvest less than 750 acres have one combine harvester. This harvester distribution is motivated by the fact Salassi and Barker (2008) found the average number of combines to be 1.5. Each combine harvester takes approximately 45 minutes to fill a load. So, the time to harvest a load in the farm with one harvester is 45 minutes plus a small random component (chosen from uniform random between negative 5 and positive 5) and the time to harvest a load in the farm size plus a small random component (chosen from uniform random between negative 2.5 (Barker, 2007, Salassi and Barker, 2008). In total, we have 456 farms in 85 zip codes with a daily capacity of 4044 loads. The number of farms in the respective counties and the location of the mills are shown in Figure 2. Each dot represents the location of a mill and the number inside a county is the number of farms in the county. The instances are available from http://http://ir.uiowa.edu/tippie\_pubs/64.

To create mill assignments for each farm, we solve a capacitated assignment problem. The objective is to minimize the sum of the distances between the farms and the mill that serves the respective farms. We assume the mills are of approximately the same size, each receiving between 365 and 370 loads. Table 1 provides the summary of the 11 farm scenarios. The first mill is served by 55 farms and has a daily quota of 370 loads and so forth.

Mill Area	# of farms	# of loads
1	55	370
2	66	367
3	29	369
4	39	365
5	23	365
6	69	370
7	28	370
8	53	370
9	27	365
10	26	365
11	41	368

Table 1: Distribution of farms and total loads



Figure 2: Distribution of farms and the mills in Louisiana

### 5.2 Computational Results

In this section, we present the results of our computational experiments with the newly created datasets. In our first test, presented in section 5.2.1, we demonstrate that simply controlling the start of the harvest time can have a remarkable effect in reducing the number of trucks needed to serve the loads at their ready times. In section 5.2.2, we explore the impact on solution quality when we reduce the size of the time blocks from an hour to half an hour to 15 minutes. This experiment also demonstrates the limits of the proposed MIP.

#### 5.2.1 Changing the Load Arrival Limits

The industrial structure considered in this paper (multiple, independent producers rather than a vertically integrated system) precludes coordinating the harvest by monitoring and varying the harvest rates at individual farms as is done in the vertically-integrated Brazilian sugarcane industry. Salassi and Barker (2008) and Salassi et al. (2009) conjecture that mill operators could coordinate harvests simply by controlling the starting times of harvests at the farms that supply the mill. In this section, we demonstrate the value of

this coordination. We consider hourly load limits of 29,30, 31, 32, 35, and 40. A limit of 29 represents the smallest integer limit that is equal across all hours and offers the chance of a solution meeting the mill's total requirement. Our results will show that increasing the hourly load limit allows solutions in which load arrivals are less evenly spread and hence mimics an increasingly uncoordinated solution.

			Arrival	Limits		
Mill Area	29 loads	30 loads	31 loads	32 loads	35  loads	40 loads
1	32	39	40	45	48	59
2	36	40	41	43	51	64
3	31	31	37	39	42	57
4	34	37	39	40	54	66
5	35	40	36	49	47	86
6	48	53	55	55	61	72
7	30	32	34	34	50	68
8	35	39	41	43	46	47
9	38	41	40	45	49	79
10	28	32	33	35	41	70
11	33	33	36	40	48	58

Table 2: Relationship between load arrival limits and number of trucks needed

Table 2 presents the number of trucks (determined in phase two of the procedure) needed to pick up all the loads at the ready times corresponding to optimal solutions of the phase one MIP solved for each of the 11 instances. As the arrival limits increase, we see that the number of trucks needed to serve a load is increasing. This increase results from the fact that, as the load limit increases, it is possible to have solutions for which the loads are less evenly spread. Thus, the results show that the number of trucks can be reduced by forcing the load arrivals to be more evenly spread across each time block. Further, the only variable that we need to control to achieve these tight load limits is the start of harvests at the farms can have a remarkable impact on the number of trucks. The differences between mills for each load limit are the result of the differences in the geographic dispersion of farms around each mill.

#### 5.2.2 Size of Time Blocks

In this section, we seek to demonstrate that reducing the size of the time blocks considered can lead to improved solutions in terms of the number of trucks required. To make a fair comparison among the solutions using various time blocks, we want to have the total number of loads be divisible by the number of one hour, half an hour, and 15 minute time blocks. For our instances, we have 13, 26 and 52 blocks with one hour, half an hour, and 15 minute blocks. The first number greater 370 such that we have a whole number as target in each block is 416. With the total loads of 416, we have an arrival limit of 32 loads, 16 loads and 8 loads, respectively, for one hour, 30 minutes and 15 minute blocks. To make a scenario with 416 loads, we take scenario 1, which has 55 farms and 370 total loads, and add one load each to the first 46 growers. Scenario 2 has 29 farms and 369 loads. So, we add two loads each to the first 18 farms and one load each to the remaining 11 farms. We proceed in a similar fashion for the remaining nine instances, thus creating 11 scenarios.

We solve these 11 scenarios for one hour, half an hour, and 15 minute blocks. To find the number of trucks, we assume a FIFO queue at the mill and that growers start harvesting at the optimal start times given by the model. Then, we assign the trucks to pick up the loads at their respective ready times. The trucks pick up the cane, travel to the mill, wait in the queue, unload and become available for next dispatch when the can is unloaded. We assume the unloading time for each load is 1.875 minutes, because the arrival window is 780 minutes and each instance has 416 loads.

In Table 3, we report the solution time for one hour, half hour, and 15 minute blocks for the 11 instances with 416 loads and the number of trucks needed to meet the ready times for the corresponding best solution. The instances marked with "\*" are not solved to optimality, and we present the number of trucks or the best solution obtained at 3600 seconds. All but one of the one hour block instances were solved to optimality. The solutions obtained using smaller time blocks better spread the load across the arrival window which in turn reduces the number of trucks needed to haul all the loads. This seems to be consistent even when the solutions obtained using smaller time blocks are not optimal. We note that, while some of the instances with 30 and 15 minute blocks were not solved to optimality in the time limit, the best found solution was still better than those of the larger time block.

To better understand the advantage of the narrower time blocks, in Table 4, we describe Scenario 1's one hour, half hour, and 15 minute block solutions. We divide the arrivals in each solution into 52 different 15-minute bins. All but the last bin (7:45 p.m. - 8:00 p.m.) is half-open. In other words, in counting the total arrivals in the first bin (7:00 a.m. - 7:15 a.m.), we assume the bin is the interval [7:00 a.m. - 7:15 a.m.) and thus include arrivals 7:00 a.m., but exclude the arrivals at 7:15 a.m. The last bin, however, is

[7:45 p.m. - 8:00 p.m.], which includes 8:00 p.m. This differs from how time blocks are defined in the MIP above. For the MIP with 15 minute blocks, the arrival at 7:15 a.m. can be counted in the first block (7:00 a.m. -7:15 a.m.) or the second block (7:15- 7:30 a.m.). More, specifically, if we have two arrivals at 7:15, one arrival could be counted towards the first block and the second arrival could be counted towards the second block. Not including the half-open intervals in the model maintains a continuous solution space, reducing computation time. We make the change in the presentation in Table 4 to simplify the presentation of the table. Because of this modeling choice, in the first 4 bins, (7:00 a.m. - 7:15 a.m., 7:15 a.m. - 7:30 a.m., 7:30 a.m., 7:30 a.m., 7:45 a.m., 7:45 a.m. - 8:00 a.m.), we have only 23 arrivals (5+3+8+7), even though the instance with hour long blocks is solved to an objective of zero. In the optimal solution, there are nine arrivals at 8:00 a.m. that the MIP counts for the block (7:00 a.m. - 8:00 a.m.).

Analysis of the arrivals in each of the 15-minutes bins offers a clue as to why the number of trucks decreases for instances run with the smaller 15-minute blocks. Having divided the arrivals for each solution into bins, we calculate the standard deviation of arrivals across the 52 bins for the one hour, half hour and 15 minute blocks, respectively, obtaining 3.02, 2.03 and 1.20, respectively. As queueing analysis predicts, the queueing at the mill increases as the variability in arrivals increases. Because of the increased queueing, more trucks are then required to serve the loads at their ready times.

# 6 Conclusion

In this paper, we seek to minimize the number of trucks needed to transport just harvested agriculture product from field to storage or processing. We do this by reducing congestion at the delivery location through the coordination of harvest start times at the individual farms. We present a two-phase solution approach that decomposes the problem by first setting the starting times of harvests at the farms and then determining the number of trucks and their assignments to loads. The first phase relies on a novel adaptation of a technique commonly used in the piecewise linearization of nonlinear functions. This linearization makes it tractable to place the harvest completion time of loads anywhere within a discrete block of time. Given the solution to the first phase, we present an algorithm to assign trucks to loads and prove its optimality.

Our computational results demonstrate that the first phase is capable of solving real-world size problems.

Further, our results demonstrate that the proposed method solve problems in which the tightly constrain the number of loads that can be placed in each discrete time block. Managerially, we demonstrate that, the more evenly the loads are spread throughout the day, the fewer trucks are needed. We do this in two ways. First, we spread the loads by forcing even numbers of loads into each time block. Second, we decrease the size of the time blocks, and in doing so, demonstrate that the number of trucks needed to serve the loads decreases in the size of the discrete time blocks used in an instance.

There are three areas of future work. A first area of research would be to explore valid inequalities or other enhancements that improve tractability. The proposed model demonstrates an ability to solve problems with tighter bounds on the number of loads per each time block than is currently found in the literature. Further, solutions to problems with short time blocks indicate that value of modeling such problems. However, Table 3 demonstrates that small time blocks with tight load limits per block are still difficult to solve to optimality. Second, as is demonstrated in the results, even small-sized blocks have some variability in arrivals and importantly in the time between arrivals. Ideally, arrivals throughout the day would be equally spaced. Preliminary work showed such a model to be intractable. In future work, we propose to explore valid inequalities and cuts for such a model in the hope of obtaining tractability. It is notable that the model proposed in this paper can be used to generate initial feasible solutions for the cases of equally spacing arrivals. A final area of future work rests in methods for managing real-time harvest logistics. The proposed model offers a planning tool for determining the start times of harvests, but is less helpful in handling the unknown events that are certain to arise throughout a day's operations.

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			# Trucks needed
			Unloading time
Scenario	Time blocks	Solution time (secs)	$= 1.875 \ minload$
1	Hour blocks	88	48
	30 minute blocks	1922	37
	15 minute blocks	*	34
2	Hour blocks	272	41
	30 minute blocks	2514	33
	15 minute blocks	*	32
3	Hour blocks	989	33
	30 minute blocks	*	31
	15 minute blocks	*	33
4	Hour blocks	52	40
	30 minute blocks	*	39
	15 minute blocks	*	38
5	Hour blocks	*	58
	30 minute blocks	*	52
	15 minute blocks	*	51
6	Hour blocks	118	52
	30 minute blocks	3279	50
	15 minute blocks	*	48
7	Hour blocks	237	35
	30 minute blocks	*	35
	15 minute blocks	*	34
8	Hour blocks	94	43
	30 minute blocks	*	37
	15 minute blocks	*	31
9	Hour blocks	1473	42
	30 minute blocks	*	41
	15 minute blocks	*	38
10	Hour blocks	509	34
	30 minute blocks	*	34
	15 minute blocks	*	32
11	Hour blocks	361	41
	30 minute blocks	3410	30
	15 minute blocks	*	32

Table 3: Relationship between size of time blocks and number of trucks needed

Table 4: Break-down of arrivals

7:00 a.m 7:15 a.m. $5$ $7$ $6$ $7:15 a.m 7:30 a.m.$ $3$ $5$ $7$ $7:30 a.m 7:45 a.m.$ $8$ $10$ $8$ $7:45 a.m 8:00 a.m.$ $7$ $7$ $8:00 a.m 8:15 a.m.$ $10$ $8$ $9$ $8:15 a.m 8:30 a.m.$ $8$ $9$ $8:15 a.m 8:30 a.m.$ $8$ $9$ $8:30 a.m 8:45 a.m.$ $13$ $9$ $10$ $8:45 a.m 9:00 a.m.$ $5$ $9$ $7$ $9:00 a.m 9:15 a.m.$ $8$ $8$ $9:15 a.m 9:30 a.m.$ $14$ $5$ $8$ $9:30 a.m 9:45 a.m.$ $8$ $12$ $9$ $9:45 a.m 10:00 a.m.$ $3$ $7$ $8$ $10:00 a.m 10:15 a.m.$ $17$ $8$ $8$ $10:30 a.m 10:45 a.m.$ $9$ $13$ $8$ $10:45 a.m 11:00 a.m.$ $5$ $5$ $8$ $11:00 a.m 11:15 a.m.$ $11$ $6$ $7$ $11:15 a.m 11:30 a.m.$ $10$ $8$ $6$ $11:30 a.m 11:45 a.m.$ $6$ $9$ $8$ $11:45 a.m 12:00 p.m.$ $5$ $6$ $6$ $12:30 p.m 12:15 p.m.$ $13$ $7$ $8$ $12:00 p.m 1:15 p.m.$ $11$ $10$ $6$ $1:30 p.m 1:230 p.m.$ $6$ $7$ $9$ $1:30 p.m 1:245 p.m.$ $9$ $7$ $2:45 p.m 2:00 p.m.$ $6$ $7$ $9$ $2:30 p.m 2:15 p.m.$ $11$ $12$ $10$ $2:45 p.m 2:30 p.m.$ $3$ $9$
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11:30 a.m. $11:45$ a.m. $16$ $9$ $8$ $11:30$ a.m. $11:45$ a.m. $6$ $9$ $8$ $11:45$ a.m. $12:00$ p.m. $5$ $8$ $8$ $12:00$ p.m. $12:15$ p.m. $13$ $7$ $8$ $12:15$ p.m. $12:30$ p.m. $5$ $6$ $6$ $12:30$ p.m. $12:45$ p.m. $11$ $10$ $10$ $12:45$ p.m. $1:200$ p.m. $6$ $7$ $6$ $1:00$ p.m. $1:15$ p.m. $11$ $10$ $6$ $1:15$ p.m. $1:30$ p.m. $6$ $7$ $9$ $1:30$ p.m. $1:45$ p.m. $9$ $11$ $7$ $1:45$ p.m. $2:00$ p.m. $6$ $5$ $7$ $2:00$ p.m. $2:15$ p.m. $11$ $12$ $10$ $2:15$ p.m. $2:30$ p.m. $3$ $3$ $9$ $2:30$ p.m. $2:45$ p.m. $12$ $9$ $7$ $2:45$ p.m. $3:00$ p.m. $6$ $7$ $9$ $3:00$ p.m. $3:15$ p.m. $8$ $8$
11:45 a.m 12:00 p.m.       5       8       8         12:00 p.m 12:15 p.m.       13       7       8         12:15 p.m 12:30 p.m.       5       6       6         12:30 p.m 12:45 p.m.       4       12       10         12:45 p.m 12:30 p.m.       6       7       6         12:00 p.m 12:45 p.m.       4       12       10         12:45 p.m 1:00 p.m.       6       7       6         1:00 p.m 1:15 p.m.       11       10       6         1:15 p.m 1:30 p.m.       6       7       9         1:30 p.m 1:45 p.m.       9       11       7         1:45 p.m 2:00 p.m.       6       5       7         2:00 p.m 2:15 p.m.       11       12       10         2:15 p.m 2:30 p.m.       3       3       9         2:30 p.m 2:45 p.m.       12       9       7         2:45 p.m 3:00 p.m.       6       7       9         3:00 p.m 3:15 p.m.       8       8       8
11:10 a.m.       12:10 p.m.       12:15 p.m.       13       7       8         12:10 p.m.       12:15 p.m.       13       7       8         12:15 p.m.       12:30 p.m.       5       6       6         12:30 p.m.       12:45 p.m.       4       12       10         12:45 p.m.       12:45 p.m.       4       12       10         12:45 p.m.       1:00 p.m.       6       7       6         1:00 p.m.       1:15 p.m.       11       10       6         1:15 p.m.       1:30 p.m.       6       7       9         1:30 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       8       8       8       8
12:10 p.m.       12:10 p.m.       10       1         12:15 p.m.       12:30 p.m.       5       6       6         12:30 p.m.       12:45 p.m.       4       12       10         12:45 p.m.       12:45 p.m.       4       12       10         12:45 p.m.       1:00 p.m.       6       7       6         1:00 p.m.       1:15 p.m.       11       10       6         1:15 p.m.       1:30 p.m.       6       7       9         1:30 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       3:15 p.m.       8       8       8
$\begin{array}{cccccccccccccccccccccccccccccccccccc$
12:45 p.m.       1:00 p.m.       6       7       6         12:45 p.m.       1:00 p.m.       6       7       6         1:00 p.m.       1:15 p.m.       11       10       6         1:15 p.m.       1:30 p.m.       6       7       9         1:30 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       3:15 p.m.       8       8       8
12:40 p.m.       1:00 p.m.       1:00 p.m.       0       1       0       0         1:00 p.m.       1:15 p.m.       11:15 p.m.       11       10       6         1:15 p.m.       1:30 p.m.       6       7       9         1:30 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       3:15 p.m.       8       8       8
1:00 p.m 1:13 p.m.       11       10       0         1:15 p.m 1:30 p.m.       6       7       9         1:30 p.m 1:45 p.m.       9       11       7         1:45 p.m 2:00 p.m.       6       5       7         2:00 p.m 2:15 p.m.       11       12       10         2:15 p.m 2:30 p.m.       3       3       9         2:30 p.m 2:45 p.m.       12       9       7         2:45 p.m 3:00 p.m.       6       7       9         3:00 p.m 3:15 p.m.       8       8       8
1:30 p.m.       1:30 p.m.       1:30 p.m.       1       7         1:30 p.m.       1:45 p.m.       9       11       7         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       3:15 p.m.       8       8       8
1:45 p.m.       1:40 p.m.       5       11       1         1:45 p.m.       2:00 p.m.       6       5       7         2:00 p.m.       2:15 p.m.       11       12       10         2:15 p.m.       2:30 p.m.       3       3       9         2:30 p.m.       2:45 p.m.       12       9       7         2:45 p.m.       3:00 p.m.       6       7       9         3:00 p.m.       3:15 p.m.       8       8       8
1:30 p.m.     2:30 p.m.     0     0     1       2:00 p.m.     - 2:15 p.m.     11     12     10       2:15 p.m.     - 2:30 p.m.     3     3     9       2:30 p.m.     - 2:45 p.m.     12     9     7       2:45 p.m.     - 3:00 p.m.     6     7     9       3:00 p.m.     - 3:15 p.m.     8     8     8
2:00 p.m 2:13 p.m.       11       12       10         2:15 p.m 2:30 p.m.       3       3       9         2:30 p.m 2:45 p.m.       12       9       7         2:45 p.m 3:00 p.m.       6       7       9         3:00 p.m 3:15 p.m.       8       8       8
2:30 p.m.     2:30 p.m.     3     5     5       2:30 p.m.     2:45 p.m.     12     9     7       2:45 p.m.     3:00 p.m.     6     7     9       3:00 p.m.     3:15 p.m.     8     8     8
2:45 p.m.         2:45 p.m.         12         5         7         9           3:00 p.m.         3:15 p.m.         8         8         8
2:49 p.m 3:00 p.m. 0 7 3 3:00 p.m 3:15 p.m. 8 8 8
0.0000.0000 = 0.0000.000000000000000000
3.15  pm $3.20  pm$ $8$ $7$ $6$
3.30  p.m. = 3.45  p.m. 6 0 6
3:45  p.m. - 5.49  p.m. = 0
$\frac{4.00 \text{ pm}}{4.15 \text{ pm}} = 0 \qquad 7 \qquad 8$
4.00  p.m. - 4.10  p.m. = 5 - 7 = 0
$4.10 \text{ a.m.} - 4.50 \text{ a.m.} = 0 \qquad 5 \qquad 5$
4:45  p.m. = 5:00  p.m. = 7 = 7 = 7
$\frac{-4.40}{5:00} \text{ p.m.} = 5:15 \text{ p.m.} \qquad 0 \qquad 11 \qquad 0$
5.00 p.m $5.10$ p.m $5$ 11 $3$
5:30  p.m. = 5:45  p.m. = 0 8 10
5.30 p.m $5.40$ p.m. $3$ $0$ $10$
6:00  p.m. 6:15  p.m. 12 0 0
6.15  pm - 6.30  pm = 8 7 0
6:30  pm = 6:45  pm = 9
6.45  pm = 7.00  pm = 5 7 7
7.00  pm = 7.15  pm = 11
7:15 p.m 7:30 p.m. 9 6 9
7:30 p.m 7:45 p.m. 6 8 9
7:45 p.m 8:00 p.m. 6 10 10