EFFORT, RACE GAPS, AND AFFIRMATIVE ACTION: A GAME-THEORETIC ANALYSIS OF COLLEGE ADMISSIONS

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ABSTRACT. I investigate the effect of Affirmative Action in higher education on both study effort choice and college placement outcomes for high school students. I model college admissions as a Bayesian game where heterogeneous students compete for seats at colleges and universities of varying prestige. There is an allocation mechanism mapping each student's achieved test score into a seat at some college. A color-blind mechanism ignores race, whereas Affirmative Action mechanisms may give preferential treatment to minorities in a variety of ways. The particular form of the mechanism determines how students' study effort is linked with their payoff, playing a key roll in shaping their behavior.

I use the model to evaluate the ability of a given college admission policy to minimize racial academic gaps—the achievement gap and the college enrollment gap—and to promote academic achievement overall. I also compare alternative policies against one another in terms of these criteria; namely, color-blind admissions, quotas, and admission preferences. These policies have very different effects on effort, achievement gaps, and allocation of college admissions in equilibrium. Compared to color-blind allocations, a simple admission preference such as the one previously used in undergraduate admissions at the University of Michigan is unambiguously detrimental to effort incentives, and is ineffective at helping minorities gain admittance to better colleges. A Quota performs better than a Michigan-style admission preference, with some positive and some negative effects on effort and achievement gaps. By design, a quota eliminates the college enrollment gap completely. Both policies widen the achievement gap among the best and brightest students.

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1. INTRODUCTION

Starting with President John F. Kennedy, the United States government has mandated Affirmative Action (AA) policies in various areas of the economy, including education, employment and procurement. The objective of AA, as articulated by policymakers, is to counteract competitive disadvantages for racial minorities due to past institutionalized racism. As President Lindon Johnson stated in his commencement address at Howard University in June, 1965,

> "You do not take a person who, for years, has been hobbled by chains and liberate him, bring him up to the starting line of a race and then say, 'you are free to compete with all the others,' and still justly believe that you have been completely fair... We seek not just freedom but opportunity."

Two persistent academic disparities among race groups are often cited as a rationale for AA in college admissions. The first is a widely documented phenomenon known as the *achievement gap*, and is typically measured in terms of standardized test scores. In 1996, the median SAT score among minority college candidates was at the 22^{nd} percentile for non-minorities.[1]

The second academic disparity, which I shall refer to as the *enrollment gap*, has to do with placement outcomes in post-secondary education: among students who attend college, minorities are under-represented at selective institutions and over-represented at low-tier schools.[2] Using institutional quality measures for American colleges, Hickman [12] shows that minorities made up almost 18% of all new college freshmen in 1996, but they accounted for only 11% of enrollment at schools in the top quality quartile. In the bottom quartile, minorities accounted for nearly 30% of enrollment.[3] These circumstances are viewed by many as residual effects of past social ills, and race-conscious college admission policies have been targeted toward addressing the problem.

Despite its intentions, much debate has arisen over the possible effects of AA on the incentives for academic achievement. Supporters claim that it levels the playing field, so

[1]Here, the working definition of the term "minority" is the union of the following three race classifications: Black, Hispanic and American-Indian/Alaskan Native. See Hickman [12] for a more detailed discussion of the figures given here. An extensive study of the black-white test score gap is given in Jencks and Phillips [14].

[2]Ultimately, policy-makers care about AA because of persistent racial wage gaps. These wage gaps are related to college admissions in two ways: first, relatively few minorities enroll in college, and second, among minority college matriculants, relatively few end up at elite institutions. Although both are interesting aspects of the college admissions problem, my notion of the enrollment gap focuses solely on the second and concerns college placement outcomes *conditional on participation in the college market*. The implications of AA for college enrollment decisions is left for future research.

[3]Institutional quality measures are based on data and methodology developed by US News & World Report for its annual *America's Best Colleges* publication. For a more detailed discussion, see Hickman [12].

to speak. The argument is that AA motivates minority students to achieve at the highest of levels by placing within reach seats at top universities—an outcome previously seen by many as unattainable. [4] In this way, it makes costly effort investment more worthwhile for the beneficiaries of the policy. Critics of AA argue just the opposite: by lowering the admission standards for college applicants, AA creates adverse incentives for them to exert less effort in competition for a seat at a given college. By making academic performance less important for one's outcome, they argue, AA creates a tradeoff between equality and achievement. Some critics of AA go even further, bringing into question whether such policies are capable of improving outcomes for disadvantaged market players, or whether the benefits go disproportionately to economically privileged members of the targeted demographic group. Sowell [5]

While the arguments on both sides of the debate seem intuitively plausible, satisfying answers to the AA controversy require an economic framework that allows for rigorous quantification of the social costs and benefits involved. With that in mind, I propose such a model of college admissions in order to better inform the policy debate. I frame the model as a Bayesian game, where heterogeneous students compete in grades for seats at post-secondary institutions. Each student is characterized by a privately-known type that determines the marginal cost of working to achieve a grade. Students observably belong to different demographic groups, and I allow for costs to be asymmetrically distributed across groups.[6] For any student who wishes to go to college, there is a seat open at some institution, but no two seats are equally desirable. Allocations of college seats are determined by a mechanism that maps each student's grade into a college seat. The mechanism may include race as a consideration. Under the payoffs induced by a given allocation rule, students optimally choose effort, based on their own type and competition they face from other students.

This model of competition for college admissions is strategically equivalent to a multi-object all-pay auction with incomplete information. Using analytic tools from auction theory, I solve for equilibrium behavior in order to assess the implications of different admission policies. Grade distributions are the equilibrium objects of principal interest from a policy standpoint, as they allow for a complete characterization of the enrollment gap, the achievement gap, and overall academic performance.

[4] Fryer and Loury [9] put forth this argument as a possible rebuttal to their "Myth 3: Affirmative Action Undercuts Investment Incentives."

[5] [21] expounds this argument in considerable detail; an extensive discussion of the opposite viewpoint is offered by Bowen and Bok [3].

[6] My intention is *not* to suggest that asymmetry reflects differences of inherent ability across differing demographic groups. The appropriate interpretation involves asymmetry arising from socioeconomic factors which affect a student's academic competitive edge. See Section 2 for a full discussion.

A meaningful investigation must encompass settings where the number of competitors is large, but analysis of the game becomes unwieldy even for moderately sized sets of players.[7] I show that there is a well-defined and simple notion of a limiting strategic environment as the number of players grows without bound. The model equilibrium can be approximated to arbitrary precision for a large enough set of competitors by treating agents and prizes as continua, rather than finite sets.

A novel feature of the model is that it allows for comparisons of alternative implementations of AA, whereas the previous literature has focused primarily on color-blind versus race-conscious college admissions. I study and compare three canonical classes of AA. The first is a quota rule, where seats are reserved for allocation to minorities, effectively splitting the competition into two separate competitions. The second variety is American-style AA, commonly referred to as an "admission preference," where minority achievement is assessed a markup before deciding who gets to attend which school. Finally, I also consider a color-blind admission rule where no preferential treatment is given.

My objective is to address four research questions. *(i)* What effect does AA have on effort incentives: does it encourage students to study more or less, and does it affect all students' effort decisions in the same way? *(ii)* What effect does it have on the achievement gap: does it widen or narrow the difference in achievement across demographic groups? *(iii)* How effective is AA at achieving proportional enrollment in college? In other words, does the intended allocative effect of a policy remain after factoring in the behavioral response produced by altering the rules of the competition? And finally, *(iv)* are there differences among alternative AA policies in terms of criteria *(i)*-*(iii)*?

Although a complete policy analysis is difficult for a researcher who cannot observe the social choice function, by making some light assumptions on the preferences of the policy-maker, one can still inform the debate in meaningful ways. Henceforth, I assume that the policy-maker has the following three objectives in selecting an admission policy: *(1)* narrowing the enrollment gap (*i.e.*, achieving parity in the profiles of colleges attended by different groups) *(2)* narrowing the achievement gap (*i.e.*, achieving parity in the profiles of academic achievement produced by different groups); and *(3)* preserving incentives which encourage high academic performance. My theoretical framework is useful for this purpose, because the equilibrium grade distributions for each group are sufficient to gauge success along each objective. I make no assumptions on how the

[7]Typically, actual college markets involve enough competitors to make this a problem. For example, the US National Center for Education Statistics reported that in 2005 over 1.8 million recent high-school graduates enrolled in college.

policy-maker weights the three objectives, so establishing a preference ranking between two policies will only be possible if one performs better along all 3 criteria.

The main contribution of this paper is in highlighting the importance of comparisons across different AA formats. Certain implementations indeed perform poorly; for example, a simple fixed grade markup formerly used at the University of Michigan. Relative to color-blind admissions, a Michigan rule erodes effort incentives for minorities by uniformly subsidizing grades regardless of individual achievement. Rational students take at least some of the grade boost as a direct utility transfer, rather than using it to bolster their competitive edge. A Michigan rule also creates discouragement effects for non-minorities which diminish their performance as well. Moreover, in equilibrium the policy merely re-shuffles admissions at the lowest-ranked colleges, leaving allocations of the best seats unchanged from a color-blind outcome. Thus, a uniform grade boost accomplishes little, but comes at a potentially high cost. More general admission preference rules can be designed to overcome some of the drawbacks of the Michigan rule.

Intuitively, minorities in a color-blind competition compete on the margin with non-minority counterparts of the same type. When a markup is assessed, they may end up competing with counterparts of higher ability levels, so students adjust behavior to the point at which they compete on the margin with students on the same competitive standing. The main shortcoming of a Michigan rule is that the marginal bonus for a little extra effort is zero, leading to the unambiguously negative behavioral response. A more sophisticated admission preference can do much better when a higher level of achievement results in a higher grade bonus. This can provide incentives for minority students to increase achievement and compete with non-minority counterparts of higher ability.

This paper also produces new contributions to the policy debate by showing that there are meaningful ways in which both the advocates and critics of AA are correct. On the one hand, a tradeoff between equality and effort does exist, in the sense that there is always some segment of the population for which achievement diminishes under AA. Another shortcoming common to all forms of AA is that the achievement gap widens among top students, relative to color-blind allocations. Moreover, certain AA policies can be ineffective at producing intended changes to market outcomes. On the other hand, some varieties of AA can indeed overcome discouragement effects for disadvantaged minorities. Some varieties of AA can produce an increase in average achievement within the minority group, and even among the population as a whole. Moreover, it is possible to achieve academic performance gains while producing a more representative college admissions profile.

The theory indicates that a definitive policy analysis is an empirical question. The final contribution of this work is in developing a theoretical AA model that allows for quantitative comparisons between competing admission rules. This is an advantage of the auction-like framework of academic competition: it provides access to a powerful set of empirical tools. Using methods developed in the auctions literature, Hickman [12] semiparametrically estimates the structural elements of the model using data on US colleges and college entrance test scores. The empirical exercise gives rise to a set of counterfactual experiments to compare market outcomes under existing AA policies with outcomes generated by alternative admission rules not observed in the data.

The rest of this paper has the following structure: in Section 2, I briefly discuss the relation between this work and the previous literature on AA. In Section 3, I give an overview of the college competition model, and in Section 4 I introduce the solution concept of an *approximate equilibrium* which adds tractability when the number of players is large. In Section 5, I show that the maximizers of a student's limiting objective function constitutes an approximate equilibrium of the finite college admissions game when the number of competitors is large. I derive approximate equilibria under colorblind admissions, quotas and admission preferences. In Section 6, I derive qualitative comparisons of achievement and race gaps under the different admissions policies for the special case where costs are linear in achievement. I also illustrate the model by solving it for a special case where private types are Pareto distributed. In Section 7, I conclude and discuss avenues for further research.

2. PREVIOUS LITERATURE

This is the first paper of which I am aware that attempts to simultaneously address quesions *(i)*–*(iv)* posed in Section 1, but tere are various papers in the literature which attempt to address some subset of questions *(i)*–*(iii)*. Coate and Loury [5] study a bilateral matching model of skills acquisition in order to address questions *(i)* (effort/human capital investment decisions) and *(ii)* (achievement gaps). Minority job applicants strategically interact with potential employers who have tastes for racial discrimination *à la* Becker, and workers decide whether to forego a fixed exogenous skill-acquisition cost. There are exactly two levels of achievement, namely, being "qualified" or "unqualified." The government mandates a minimal minority employment level—similar to a quota in the current model—and skill investment decisions are given by a threshold rule: all workers with costs below a fixed cutoff choose to acquire skills. The threshold rule is such that the government mandate can be gradually increased over time so that all workers previously acquiring skills still choose to do so, and an additional set of minority workers also choose to acquire skills.

An important difference between Coate and Loury [5] and this paper is the agents' choice set. In the former model, agents face a binary choice of whether to acquire a fixed skill level at a fixed, exogenous cost. In contrast, I allow for agents to choose any skill level, meaning that the exact cost incurred is at the agent's discretion. Heterogeneity among individuals exists in the form of differences among marginal costs of skill acquisition. When this is true, any AA policy changes every player's behavior. This creates a tradeoff between equality and effort, and policy changes may no longer be unambiguously desirable.

However, there is a more fundamental distinction between bilateral matching models in general and the all-pay auction framework. The value added in the theoretical approach I employ is that it includes the competitive interaction between two different groups that are unevenly affected by AA. As it turns out, this is a central concern when assessing how behavior responds to incentive changes under a given policy. For example, in a color-blind world with heterogeneous students, each one intuitively competes on the margin with others of similar ability levels. However, when the test scores of one group are assessed a markup—as in an admission preference—it may be possible that minority students end up competing on the margin with non-minorities of differing ability levels. As I show later on, a common theme arising from such policies is that students adjust their behavior so that on the margin they compete against other students who are on roughly the same competitive standing. This concept informs the researcher on the properties of markup functions that produce desired changes (*e.g.*, the marginal markup for an additional unit of achievement).[8]

Various models from the contests literature also attempt to address questions *(i)* and *(ii)*. Two such papers are Fain [6] and Fu [11] (see also [10]). Both are two-player all-pay contests with complete information (*i.e*, where heterogeneity among competitors is commonly observable). Both models study an interaction between one advantaged player and one disadvantaged player competing for a single prize, and both find that an admission-preference-like AA rule benefitting the disadvantaged player increases effort exerted by both players. They then use these results to argue that colleges will admit a higher-quality body of students if the school gives preference to the minority students by weighting their grades more heavily. Schotter and Weigelt [20] perform an experimental analysis of a two-player model similar to Fain [6], with similar results.

However, in extrapolating their results to a competition involving many students, these authors implicitly assume that *every* beneficiary of the AA policy is at a competitive disadvantage to *every* other student not benefitting from it. However, this assumption is

[8]Note to self: mention Moro and Norman (2003) here.

inappropriate in the context of college admissions, where AA is based only on one's observable race, rather than one's unobservable characteristics which determine academic competitiveness. The current model produces very different results, due to the fact that there are both high-cost types and low-cost types in the minority group, all of whom benefit from AA. There are also high-cost types in the non-minority group who do not benefit from AA.

In January of 2008, presidential candidate Barack Obama famously stated in a television interview that his daughters should not be treated as disadvantaged in college admissions decisions, and that perhaps white children raised in poverty should benefit from AA. The results of this paper are consistent with the intuition behind Mr. Obama's assertion: a common feature in both classes of AA considered here is a reduction in effort among both low-cost minorities and high-cost non-minorities. For the former group, AA provides a competitive boost that was not needed; for the latter, AA exacerbates discouragement effects. In short, when there are both gifted and challenged students in each demographic group, the unambiguous benefits arising from AA are no longer a foregone conclusion.

A final related paper is Franke [7], who analyzes the effect of an admission-preference-like AA policy in a contest with many players. Franke shows that when the policy-maker is fully informed on student heterogeneity, he can design a grade-weighting scheme that raises all players' effort, relative to a color-blind rule. While this is certainly an improvement over a simplistic 2-player model, Franke still relies on the strong assumption of complete information to construct the beneficial policy. In that sense, the paper can be thought of as a characterization of the "first-best" outcome, where no information is hidden from the policy-maker. By contrast, I evaluate the tradeoffs faced by a policy-maker who cannot observe individual characteristics other than race. A college admissions college board can see each student's grade, but it cannot observe the cost incurred to achieve that grade. In keeping with the *Wilson doctrine*, I constrain the current theoretical exercise to evaluating policies that are implementable without knowledge of model primitives like private types and the associated distributions.

As for question *(iii)*—characterizing the equilibrium enrollment gap—there are many models designed to characterize admissions outcomes at a single post-secondary institution under AA. The contests papers mentioned above fit this description, as they all involve competition for a single indivisible good. Another paper is Chan and Eyster [4], which studies AA in a setting where a single college chooses what profile of students to admit, subject to a capacity constraint. However, if colleges and universities differ in meaningful ways in terms of quality, these models cannot address the question of how admission policies affect racial composition among different segments of the quality spectrum. This issue requires a model where many heterogeneous college applicants are being matched with many heterogeneous colleges. In the current setup, this aspect of college admissions is captured by a set of distinct prizes for which students compete.

3. The Model

I model the competition among high-school students for college admissions as a Bayesian game. Students belong to two demographic groups—minorities and non-minorities—and each student is characterized by a privately-known study cost type. Students compete in grades for a set of heterogeneous prizes—seats at colleges/universities of differing quality—and types determine the costliness of academic achievement. Students have single-unit demands and prizes are allocated by a pre-specified mechanism, according to grades. AA enters the model if the mechanism bases allocations partially on race as well. Students can observe the set of prizes before making decisions, but they must incur a non-recoverable cost associated with academic achievement *before* learning which prize they will receive. A student's payoff at the end of the game is the utility derived from consuming a prize, minus the utility cost of his achieved grade. An equilibrium of the game is characterized by a set of achievement functions that prescribe each student's optimal effort level. A formal description of the components of the game is given below.

3.1. **Costs and Benefits.** The agents are a set $\mathcal{K} = \{1, \dots, K\}$ of students who observably belong to a minority group $\mathcal{M} = \{1, 2, \dots, M\}$ or a non-minority group $\mathcal{N} = \{1, 2, \dots, N\}$, where $M + N = K$. Students are heterogeneous, and each is characterized by a privately-known study cost type $\theta \in [\underline{\theta}, \overline{\theta}]$. Agents view the types of their opponents as independent random variables, and there is a common prior on types within each group, $\Theta \sim F_i(\theta)$, $i = \mathcal{M}, \mathcal{N}$. Students have access to a common strategy set $S = \mathbb{R}_+$, comprising grades/test scores. In order to achieve grade level s, an agent must incur a cost $\mathcal{C}(s;\theta)$, which depends on his type.

This specification of costs lends itself to several interpretations where θ could arise from either cognitive or non-cognitive skills. Costs could be reflective of an underlying labor-leisure tradeoff where students differ either by preferences for leisure, or by the amount of labor input required to produce a unit of s. Alternatively, it could reflect some psychic cost of exerting mental effort to learn new concepts, where the amount of effort required to produce a given grade differs among students. The cost type θ could also reflect many other external factors affecting students' academic performance such as home conditions, affluence, school quality, and access to things like health-care and tutors.

The rewards for academic achievement are a set of prizes

$$\mathbf{P}_{\mathcal{K}} = \{p_k\}_{k=1}^{K},$$

where p_k denotes the utility of consuming the k^{th} prize. The prizes are seats at distinct colleges and universities, and students have single-unit demands: they can only attend one school. There are enough prizes for every student who competes (*i.e.*, there are enough seats open to serve anyone who wishes to go to college), but no two prizes render the same utility: $p_k \neq p_j,\ k \neq j$. At the end of the game, an agent's payoff is the utility from consuming a prize minus the cost of achievement, or

$$\Pi(s;\theta) = p - \mathcal{C}(s;\theta).$$

The alert reader will notice that I have implicitly assumed agents have identical preferences over differing colleges and universities. However, it is not essential to the model for all students to place the same value on a seat at a given college; the important assumption here is that students rank prize values the same.[9] Without this assumption, a policy discussion concerning admission outcomes is impossible, and the researcher is left with the unsatisfying conclusion that fewer minorities attend elite institutions simply because they prefer it that way. An alternative view of the homogeneous ranking assumption is that students have similar preferences over school attributes such as per-pupil spending, graduation rates, student-faculty ratios, *etc.*

3.2. **College Admission Policies.** Grades are mapped into payoffs as the outcome of a matching market with three stages: students send reports of their achievement level to various colleges/universities, admissions boards make acceptance/rejection decisions, and students choose among the options given to them by the market. I assume that there are no frictions in the matching market, so that its outcome can be implemented by a centralized mechanism which uses the set of grades $\mathbf{s} = \{s_{\mathcal{M},1}, \ldots, s_{\mathcal{M},M}, s_{\mathcal{N},1}, \ldots, s_{\mathcal{N},N}\}$ achieved by all students to allocate prizes. In other words, the assumption here is that the market is efficient in the sense that it is effective at matching higher performing students (holding race constant) with higher quality schools.

A simple "color-blind" admission rule is one which assortatively matches prizes with grades. The student submitting the highest grade is awarded the most valuable prize, and so on. In what follows, it will be convenient to treat a competition with color-blind admissions as the baseline model.

[9] For certain specifications of the cost function (*e.g.*, $\mathcal{C}(s;\theta) = \theta h(s)$) each student's objective function can be normalized by his type to get a game where achievement is uniformly costly across all competitors, but where each derives different utility from occupying a given seat. In this equivalent model, all prizes still follow a uniform ranking, but the marginal utilities of upgrading to the next best prize are unique to each competitor.

As for AA, consider first a quota system similar to what's known as "Reservation Law" in India. This law mandates that a certain percentage of seats be set apart for allocation only to certain demographic groups. There are many possible quota rules indexed by a number $q \in \{1, 2, \ldots, M\}$ of prizes reserved for minorities. However, for simplicity I will consider only the case of a full quota rule, where exactly M prizes are reserved for minorities. Under a full quota rule, students compete only with members of their own group. It is also necessary to specify *how* prizes are selected for reservation. There are many possibilities once again, but for simplicity I will focus on the case where a *representative* set of M prizes is set aside. This can be accomplished by either randomly selecting M prizes from the set $\mathbf{P}_{\mathcal{K}}$, or it can be by first ordering prizes by quality and selecting out every m^{th} prize, where $m = \frac{M+N}{M}$. In what follows, it will be easiest to consider the random selection method, but this is without loss of generality: when the set of prizes is large, the overall effect will be the same.

The form of AA as implemented in the US is different, due to a 1978 Supreme Court ruling that explicit quotas—*i.e.*, earmarking seats for allocation only to students of a particular race—are unconstitutional. Since then, American higher education institutions have been forced to seek other means by which to implement AA. The resulting system is commonly referred to as an "admission preference," where test scores achieved by minority students are given more weight in admissions decisions. I model an admission preference rule as a grade transformation function $\tilde{S} : \mathbb{R}_+ \to \mathbb{R}_+$. This mechanism matches prizes assortatively with non-minority grades and *transformed* minority grades

$$\{s_{\mathcal{N},1}, \ldots, s_{\mathcal{N},N}, \tilde{S}(s_{\mathcal{M},1}), \ldots, \tilde{S}(s_{\mathcal{M},M})\}.$$

In other words, under an admission preference admissions boards view each minority student with a grade of s as if he had submitted a grade of $\tilde{S}(s)$ instead.[10]

Regardless of whether admissions are color-blind, or follow some form of AA, ties between competitors are broken randomly. That is, in the event of a tie between two or more scores (some of which may be transformed), each student involved in the tie is assigned a random index for the purpose of ranking him with his competitors.

Before making decisions, agents observe the set of prizes $\mathbf{P}_{\mathcal{K}}$, the admission rule, $\mathcal{R} \in \{cb$ (color-blind), q (quota), ap (admission preference)$\}$, and the number of competitors from each group M and N. As mentioned above, students share a common prior on the type distributions $F_{\mathcal{M}}$ and $F_{\mathcal{N}}$. Under the payoff correspondence $\Pi(\mathbf{s}; \theta)$ induced by a particular admission rule, students optimally choose grades based on their own type and taking into account their opponents' optimal behavior. A *(group-wise) symmetric*

[10] It should be noted here that this grade transformation rule defines a very general class of mechanisms. In fact, the color-blind and quota rules are both special cases.

equilibrium of the Bayesian game $\Gamma(M, N, \mathbf{P}_{\mathcal{K}}, \mathcal{R})$ is a set of achievement functions γ_i : $[\underline{\theta}, \overline{\theta}] \rightarrow \mathbb{R}_+$, $i = \mathcal{M}, \mathcal{N}$ which generate optimal grades, given that ones' opponents behave similarly. For the remainder of this thesis, I shall restrict attention to the class of symmetric equilibria.

3.3. **Policy Objectives.** Equilibrium achievement functions and type distributions induce a set of group-specific grade distributions, $G_{\mathcal{M}}$ and $G_{\mathcal{N}}$ and a population grade distribution G. These are ultimately the objects of interest from a policy standpoint, as they fully characterize achievement, achievement gaps and enrollment gaps in equilibrium. Henceforth, the achievement gap shall be formally represented by a function $\mathcal{A} : [0, 1] \rightarrow \mathbb{R}$ defined by

$$\mathcal{A}(q) \equiv G_{\mathcal{N}}^{-1}(q) - G_{\mathcal{M}}^{-1}(q).$$

In words, $\mathcal{A}$ characterizes the difference between minority and non-minority achievement at each quantile of the grade distributions. Thus, to eliminate the achievement gap is to accomplish an outcome where $\mathcal{A}(q) = 0, \ \forall q \in [0, 1]$.

As for the enrollment gap, let $F_{P_i}(p), \quad i = \mathcal{M}, \mathcal{N}$ denote the distribution of prizes awarded to either group in equilibrium. Then the enrollment gap is a function $\mathcal{E}$: $[0, 1] \rightarrow \mathbb{R}$ defined by

$$\mathcal{E}(q) \equiv F_{P_N}^{-1}(q) - F_{P_M}^{-1}(q).$$ [11]

Once again, to eliminate the gap is to accomplish an outcome where $\mathcal{E}(q) = 0, \ \forall q \in [0, 1]$. Finally, the overall profile of academic achievement is represented by the population grade distribution,

$$G(s) = \frac{M}{M+N} G_{\mathcal{M}}(s) + \frac{N}{M+N} G_{\mathcal{N}}(s).$$

Measures of cost and benefit cited in the policy debate over AA are often related to, or derived from $\mathcal{A}$, $\mathcal{E}$, or G. For example, a statement about the test score gap that "the median minority SAT score lags behind the non-minority median by 150 points," is equivalent to the statement $\mathcal{A}(.5) = 150$. The reason for defining race gaps and achievement in such general terms is that it avoids imposing strong assumptions on what policy-makers care about. To wit, if preferences place the same weight on the enrollment gap at every point of the college quality spectrum, then $\mathcal{E}$ could be reduced to $\mathcal{E} = \int_0^1 \left(F_{P_N}^{-1}(q) - F_{P_M}^{-1}(q) \right) dq$; but if the policy-maker cares more about the enrollment gap at elite schools, then this would be inappropriate.

[11] At the moment, this is an abuse of notation, given that F_{P_i} is a step function having no proper inverse, since the set of prizes is finite. However, in the limit as the number of prizes (and players) grows without bound, the problem disappears. The analysis hereafter will concentrate on the limiting case, as it meaningfully reflects the character of a large finite game while adding tractability to the model.

Having formalized my notion of race gaps and achievement, I shall proceed under the following light assumptions regarding the policy-maker's preferences:

Assumption 3.1. For two achievement gap functions, $\mathcal{A}^*$ and $\mathcal{A}$,

$$\mathcal{A}^*(q) \leq \mathcal{A}(q) \ \forall q \in [0,1] \quad \Rightarrow \quad \mathcal{A}^* \succcurlyeq \mathcal{A},$$

and $\mathcal{A}^* \succ \mathcal{A}$ if in addition

$$\exists q^* \in [0,1] \ \ s.t. \ \ \mathcal{A}^*(q^*) < \mathcal{A}(q^*).$$

Assumption 3.2. For two enrollment gap functions, $\mathcal{E}^*$ and $\mathcal{E}$,

$$\mathcal{E}^*(q) \leq \mathcal{E}(q) \ \forall q \in [0,1] \quad \Rightarrow \quad \mathcal{E}^* \succcurlyeq \mathcal{E},$$

and $\mathcal{E}^* \succ \mathcal{E}$ if in addition

$$\exists q^* \in [0,1] \ \ s.t. \ \ \mathcal{E}^*(q^*) < \mathcal{E}(q^*).$$

Assumption 3.3. For two population grade distributions, G^* and G,

$$G^*(s) \leq G(s) \ \forall s \in \mathbb{R}_+ \quad \Rightarrow \quad G^* \succcurlyeq G,$$

and $G^* \succ G$ if in addition

$$\exists s^* \in \mathbb{R}_+ \ \ s.t. \ \ G^*(s^*) < G(s^*).$$

3.4. **Model Assumptions.** In order to guarantee existence of a pure-strategy equilibrium, it will be necessary to make the following assumptions on the form of the study cost function:

Assumption 3.4. $\frac{\partial \mathcal{C}}{\partial s} > 0$; $\frac{\partial \mathcal{C}}{\partial \theta} > 0$; $\frac{\partial^2 \mathcal{C}}{\partial s^2} \geq 0$; and $\frac{\partial^2 \mathcal{C}}{\partial s \partial \theta} \geq 0$.

In words, costs are assumed to be convex and increasing in achievement level s and type θ. Marginal costs are also assumed to be increasing in student types, so that a smaller θ not only means a lower cost of achieving grade level s, but also a lower marginal cost of increasing output from s to $s + \varepsilon$.

It is also necessary to make the following assumptions on beliefs:

Assumption 3.5. The type distributions $F_{\mathcal{M}}(\theta)$ and $F_{\mathcal{N}}(\theta)$ have continuous and strictly positive densities $f_{\mathcal{M}}(\theta)$ and $f_{\mathcal{N}}(\theta)$, respectively.

One aspect of the model is worth highlighting here. By assuming that type distributions are static and exogenous, I am implicitly taking a short-run view of policy implications. One might conceive of a broader model in which the policy-maker designs a mechanism today so as to affect the evolution of types for future generations (*i.e.*, the children of today's college freshmen). Such an undertaking is beyond the scope of the current exercise, and is left for future research. Instead, I shall concentrate on the implications of the policy-maker's choices for actions and outcomes of older school children and today's college candidates, whose types can reasonably be viewed as fixed.

Finally, when considering an admission preference rule, I shall restrict attention to policy functions $\widetilde{S}$ satisfying certain sensibility criteria.

Assumption 3.6. $\widetilde{S}(s)$ is a strictly increasing function lying above the 45°-degree line.

Assumption 3.7. $\widetilde{S}(s)$ is continuously differentiable.

Assumption 3.6 corresponds to the notion that the policy is geared toward assisting minorities, effectively moving each minority student with a grade of s ahead of each non-minority student with a grade of $\widetilde{S}(s) \geq s$. Moreover, it states that a policy-maker will not choose to reverse the ordering of any segment of the minority population, so that some students are awarded prizes of lesser value than other students within their own group whose grades were lower. Assumption 3.7 implies that the policy-maker does not make abrupt jumps in either the assessed grade boost, or the marginal grade boost. Aside from characterizing the behavior of a sensible policy-maker, Assumptions 3.6 and 3.7 also guarantee that introducing $\widetilde{S}$ into the model does not interfere with existence of the equilibrium.

3.5. **An Auction-Theoretic View of the Game.** The model defined above is strategically equivalent to a special type of game known in the contests literature as an *all-pay auction*. An all-pay auction is a strategic interaction in which agents compete for a limited resource by incurring some type of unrecoverable cost *before* learning the outcome of the game. In the present model of college competition, high school students cannot recover lost leisure time or disutility incurred by study effort if they discover that they didn't make it into the college they had hoped for.

The centralized college admissions board is analogous to an auctioneer selling off a set of heterogeneous prizes according to a pre-determined mechanism. Students are similar to bidders, and the grades they work for are analogous to bids. The value here in recognizing the connection to auction theory is that I can import a well-developed set of analytic tools for characterizing the equilibrium. For example, as the following proposition shows, I can conclude *a priori* that a monotonic equilibrium exists. As I will shortly demonstrate, existence and monotonicity provide an invaluable step toward analytic and computational tractability of the model when K is large, as it is in college admissions.

Proposition 3.8. *In the college admissions game $\Gamma(M, N, \mathbf{P}_{\mathcal{K}}, \mathcal{R})$ with $\mathcal{R} \in \{cb, q, ap\}$, there exists a unique symmetric pure-strategy equilibrium $(\gamma_{\mathcal{M}}(\theta), \gamma_{\mathcal{N}}(\theta))$ where achievement is strictly decreasing in types; therefore, $G_i(s) = 1 - F_i\left((\gamma_i)^{-1}(s)\right)$.*

A formal proof of Proposition 3.8 is left to an appendix, as it is fairly involved. Briefly though, existence and monotonicity is a straightforward application of results proven in Athey [2]. From there, continuity and differentiability follow directly (although the argument is somewhat complex) from the definition of an equilibrium. Once differentiability has been established, some well-known results from differential equations theory establish a unique solution to the first-order conditions of a student's decision problem. Any symmetric equilibrium prescribes optimal behavior (by definition), and therefore it must satisfy the unique solution to the first-order conditions.

4. EQUILIBRIUM ANALYSIS

4.1. **Solution Concept: "Approximate Equilibrium".** In this section I introduce an alternative solution concept that I adopt for tractability. For large $K = M + N$, the model equilibrium is analytically and computationally intractable, because a decision-maker's objective function is a complicated sum of functions based on the order statistics of opponents' costs. Agents know that their *ex-post* payoff depends on their rank within the grade distribution, and under monotonicity this is the same as their rank within the realized cohort of opponents. Thus, expected equilibrium payoffs are a weighted average of the prizes, where the weight on the k^{th} best prize is one's probability of being the k^{th} lowest order statistic among K competing types.

For simplicity and tractability, I assume that the number of competitors is large enough so that one has a very good idea of one's rank within the realized sample of competitor types. I approximate this large, finite model by considering the limiting case as $K \to \infty$, but in order to do so I must first introduce some additional notation. Let μ denote the asymptotic mass of the minority group: as each new agent is created, nature assigns him to group $\mathcal{M}$ with probability μ, and then he draws a type from the appropriate group-specific distribution. Given this assumption, with probability one the limiting sample of competitors is a dense set on the interval $[\underline{\theta}, \overline{\theta}]$, and each knows with certainty that his sample rank is the same as his rank in the unconditional type distribution $\mu F_{\mathcal{M}}(\theta) + (1-\mu) F_{\mathcal{N}}(\theta)$.[12]

[12] The fact that the limiting sample is a dense set can be seen by applying the following logic: given any two numbers $\theta, \theta' \in [\underline{\theta}, \overline{\theta}]$, where $\theta < \theta'$, the probability mass assigned to the interval (θ, θ') is strictly positive under my assumptions on F_{Θ}. Therefore, as the number of iid draws from F_{Θ} gets large, the probability of hitting the interval (θ, θ') at least once approaches one. Thus, a countably infinite random sample of agents will be everywhere dense on $[\underline{\theta}, \overline{\theta}]$.

For analytic convenience, I also assume that prizes are generated as independent draws from a compact interval $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}_+$ according to a known prize distribution $F_P(p)$ satisfying

Assumption 4.1. F_P has a continuous density $f_P(p)$, which is strictly positive on $\mathcal{P}$; and

Assumption 4.2. (*zero surplus condition*) $\underline{p} = \mathcal{C}(0; \overline{\theta})$.

Even though prize values are *ex-ante* observable, framing them in this way provides an intuitive view of the limiting set of prizes: as $K \to \infty$, $\mathbf{P}_{\mathcal{K}}$ becomes a dense set on $\mathcal{P}$, and the rank of a prize with value p converges to $F_P(p)$.

Assumption 4.2 is necessary because it provides a boundary condition that is used to solve the equilibrium equations. Although the current theoretical exercise focuses solely on the competition among high-school students for college admissions, the zero surplus condition can be thought of as reflecting broader market forces not explicitly included in model. In the broader model, prize values are the additional utility one gains from going to college versus opting out, and $[\underline{\theta}, \overline{\theta}]$ is the set of individuals who demand a college seat, being a subset of a larger group of individuals, some of whom choose the outside option. If schools and firms can freely choose to enter the market and supply either college seats or jobs for unskilled laborers, the marginal college candidate—the highest type opting for college, $\overline{\theta}$—will be just indifferent between attending college and entering the work force as an unskilled laborer. This point highlights a limitation of the current model: it attempts to characterize student behavior *conditional on participation in the post-secondary education market*, and it is not intended to provide insights into the decision of whether to acquire additional education. This aspect of the college admissions problem is left for future research.

With that out of the way, I can treat both agents and prizes as if they belong to a continuum, rather than a finite set. This allows me to avoid framing decisions in terms of the distributions of complicated order statistics, and it reduces each agent's decision problem to a simple objective function expressed in terms of θ, μ, $F_{\mathcal{M}}$, $F_{\mathcal{N}}$ and F_P. Given the well-behaved nature of the model primitives, the maximizers of the finite objective functions converge to the maximizer of the limiting objective function, which allows me to derive what I refer to as an *approximate equilibrium*.

Definition 4.3. Consider a generic game $\Gamma(\mathcal{K}, S, \Pi)$, where $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of players, $S_i \subseteq \mathbb{R}$ is the strategy space for the i^{th} player, and $\Pi(s_1, \ldots, s_K)$ characterizes payoffs on $S = S_1 \times \cdots \times S_K$. Given $\delta > 0$, a *δ-approximate equilibrium* is a K-tuple $\mathbf{s}^\delta = (s_1^\delta, \ldots, s_K^\delta)$, such that there exists an equilibrium $\mathbf{s}^* = (s_1^*, \ldots, s_K^*)$ of Γ, where $\|\mathbf{s}^\delta - \mathbf{s}^*\|_{\sup} < \delta$.

The approximate equilibrium concept is more relevant for my purposes than the ε-equilibrium introduced by Radner [18], which is a profile of strategies generating payoffs that are ε close to payoffs in some equilibrium of Γ. The drawback of an ε-equilibrium is that it need not resemble the equilibrium strategies which generate the payoffs being approximated.[13] In my case, the strategies are a principal concern: I wish to concentrate on the effects of admission policies on both payoffs *and* behavioral choices. However, there is a connection between the two concepts, as the following remark demonstrates.

Remark 4.4. For a Nash equilibrium $\mathbf{s}^*$ of Γ, if $\mathbf{s}^* \subset U \subset S$, where U is a neighborhood of $\mathbf{s}^*$, and if the payoff function Π is continuous on U, then the set of payoffs generated by ε-equilibria and δ-approximate equilibria form bases of neighborhoods of equilibrium payoffs. That is, given an ε-equilibrium associated with $\mathbf{s}^*$, there exists $\delta > 0$ such that for a δ-approximate equilibrium we have

$$\|\Pi(\mathbf{s}^{\delta}) - \Pi(\mathbf{s}^*)\|_{\sup} < \varepsilon.$$

Conversely, given a δ-approximate equilibrium associated with $\mathbf{s}^*$, there exists $\varepsilon > 0$ such that for an ε-equilibrium we have

$$\|\Pi(\mathbf{s}^{\varepsilon}) - \Pi(\mathbf{s}^*)\|_{\sup} < \|\Pi(\mathbf{s}^{\delta}) - \Pi(\mathbf{s}^*)\|_{\sup}. \square$$

In the context of the college admissions model, I seek to characterize a set of approximate achievement functions, denoted $\gamma^{\infty}_{\mathcal{M}}(\theta)$ and $\gamma^{\infty}_{\mathcal{N}}(\theta)$, such that given a fixed tolerance level $\delta > 0$, the functions approximate actual equilibrium achievement to δ-precision for large enough K.

My approximate equilibrium concept is similar to the *oblivious equilibrium* developed by Weintraub, Benkard, and Van Roy [23, WBR] to approximate Markov Perfect Equilibria in dynamic oligopoly games. In such models, firms (and researchers) must compute a complex and intractable state transition process in order to exactly determine equilibrium strategies. Instead, WBR assume that firms make nearly optimal decisions based on a long-run average industry statistic which is inexpensive to compute. This issue of computational tractability leads to an alternative interpretation of approximate equilibria. Aside from being a useful approximation of equilibrium behavior, one could also view them as an exact characterization of the behavior of agents with bounded information processing ability. Rather than precisely tracking expected payoffs based on all

[13] Radner [18] used an ε-equilibrium to resolve a dilemma in dynamic Cournot oligopoly games. For a fixed set of firms, as long as the number of periods is finite, the unique subgame perfect equilibrium involves static equilibrium strategies being played every period, whereas collusion suddenly becomes possible in the limit. Radner showed that there is a collusive ε-equilibrium of the finite-horizon Cournot game in which cartels are sustainable. Equilibrium payoffs can be replicated to arbitrary precision (*i.e.*, it is nearly optimal to collude if the time horizon is far enough away), even though the ε-equilibrium strategies are excluded from a neighborhood of equilibrium strategies.

of the order statistics of a large set of competitors, a cognitively constrained agent with type θ might find it more attractive to base decisions on his limiting rank $F_i(\theta)$ instead.

5. APPROXIMATE EQUILIBRIA UNDER AFFIRMATIVE ACTION

I shall proceed by deriving the maximizers of an agent's limiting objective function as the natural processes described in Section 4.1 generate increasingly large sets of competitors and prizes. I then prove that the resulting derivations satisfy Definition 4.3 above. From this point on, all discussion and derivations will be in terms of the approximate equilibrium, $\left(\gamma_{\mathcal{M}}^{\infty}(\theta), \gamma_{\mathcal{N}}^{\infty}(\theta)\right)$, so I shall drop the ∞ superscript for notational ease. Moreover, to avoid tedious verbosity I shall henceforth refer to the approximate equilibrium and the approximate achievement functions simply as "the equilibrium" and "the achievement functions," unless the context requires more specificity. If it becomes necessary to distinguish between the actual equilibrium of a game with K agents and the approximate equilibrium, I shall refer to the former as the "finite equilibrium" and I shall abuse the notation slightly and denote the former by $\gamma(\cdot; K)$, listing K as a parameter. Keeping in mind the processes generating agents and prizes, this notational abuse is not unreasonable: by the law of large numbers, any two randomly generated games with a large number of players K will be probabilistically very similar.

Superscripts shall be used henceforth to keep track of the admission policy which defines payoffs in the game. Under policy $\mathcal{R} \in \{cb, q, ap\}$, the achievement functions and grade distributions are denoted by $\gamma_{\mathcal{M}}^{\mathcal{R}}(\theta)$, $\gamma_{\mathcal{N}}^{\mathcal{R}}(\theta)$, $G^{\mathcal{R}}$, $G_{\mathcal{M}}^{\mathcal{R}}$, and $G_{\mathcal{N}}^{\mathcal{R}}$. Finally, in what follows it will sometimes be convenient to work with the inverse achievement function, which I denote by $\psi_i^{\mathcal{R}}(s) \equiv \left(\gamma_i^{\mathcal{R}}\right)^{-1}(s)$.

5.1. The Color-Blind Game.

Recall that a color-blind allocation rule means simple positive assortative matching of prizes with grades. I claim (proof to follow later) that in the limit, in equilibrium, this process is equivalent to using the following reward function for a student submitting a grade of s:

$$\begin{aligned}
\pi^{cb}(s) =& F_P^{-1}\left[G^{cb}(s)\right] \\
=& F_P^{-1}\left[\mu G_{\mathcal{M}}^{cb}(s) + (1-\mu) G_{\mathcal{N}}^{cb}(s)\right] \\
=& F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu) F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right].
\end{aligned}$$

Intuitively, the quantiles of the population grade distribution $G^{cb}(s)$ are mapped into the corresponding prize quantiles. Since individuals' limiting payoffs do not depend on race, it follows that $\gamma^{cb}_{\mathcal{M}}(\theta) = \gamma^{cb}_{\mathcal{N}}(\theta) = \gamma^{cb}(\theta)$; hence, the lack of subscripts on the inverse achievement functions in the third line.[14]

In equilibrium, the limiting net payoff for an agent with cost type θ submitting grade s is

$$\Pi^{cb}(s;\theta) = F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right] - \mathcal{C}(s;\theta).$$

Differentiating, I get the following FOC:

$$(1) \qquad -\frac{\mu f_{\mathcal{M}}[\psi^{cb}(b)] + (1-\mu)f_{\mathcal{N}}[\psi^{cb}(b)]}{f_P\left(F_P^{-1}\left[1 - \left(\mu F_{\mathcal{M}}\left[\psi^{cb}(s)\right] + (1-\mu)F_{\mathcal{N}}\left[\psi^{cb}(s)\right]\right)\right]\right)}\frac{d\psi^{cb}(s)}{ds} = \mathcal{C}'(s;\theta).$$

Using the fact that $\frac{d\psi^{cb}(s)}{ds} = \frac{1}{(\gamma^{cb})'(\psi^{cb}(s))}$, and the fact that in equilibrium we have $\psi^{cb}(s) = \theta$, I can substitute to get

$$(2) \qquad (\gamma^{cb})'(\theta) = -\frac{\mu f_{\mathcal{M}}(\theta) + (1-\mu)f_{\mathcal{N}}(\theta)}{f_P\left(F_P^{-1}\left[1 - \mu F_{\mathcal{M}}(\theta) - (1-\mu)F_{\mathcal{N}}(\theta)\right]\right)\mathcal{C}'[\gamma^{cb}(\theta);\theta]}.$$

This differential equation partially solves for equilibrium achievement, but a boundary condition is also needed.

By monotonicity, a student with cost type $\overline{\theta}$ is sure to be awarded the lowest quality prize, so the Assumption 4.2 implies the following boundary condition:

$$(3) \qquad \gamma^{cb}(\overline{\theta}) = \mathcal{C}^{-1}(\underline{p};\overline{\theta}).$$

With that, I am ready to prove that the derivations above provide meaningful insights into the equilibrium of a finite college admissions games where the number of competitors is large. The proof is fairly involved, but it is based on simple ideas. I first prove that the finite objective functions converge pointwise in probability to the limiting objective function listed above. By viewing a K-player game as being randomly generated by the natural processes outlined in Section 4.1, one can think of a player's objective function as a random variable; hence the concept of convergence in probability. Using pointwise convergence, I can invoke Egorov's Theorem to deliver uniform convergence of the sequence of finite objective functions. Finally, using uniform convergence, I can invoke the theorem of the maximum to show that the finite maximizers are close to the solution of equation (2) and boundary condition (3) for large K.

[14]The theorist with experience in asymmetric auctions may find this statement puzzling, but one must keep in mind that it merely applies to *limiting* payoffs. In a two-player game, differing behavior arises from the fact that a minority and a non-minority with the same private cost type will view their likely standing in the distribution of realized competition differently, due to the asymmetry in the cost distributions. However, the likely difference between their expected ranks vanishes as the number of players gets large.

Theorem 5.1. *Given ρ, ε, $\delta > 0$, there exists $K^* \in \mathbb{N}$, and a set $E \subset [\underline{\theta}, \overline{\theta}]$ having (Lebesgue) measure $m(E) < \rho$, such that for any $K \geq K^*$, on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$ we have the following:*

(i) *$\gamma^{cb}(\theta)$ as defined by equation (2) and boundary condition (3) generates an ε-equilibrium of the K-player color-blind game, and*

(ii) *$\gamma^{cb}(\theta)$ is a δ-approximate equilibrium for the K-player color-blind game, or*

$$\|\gamma^{cb}(\theta) - \gamma_i^{cb}(\theta; K)\|_{\sup} < \delta, \quad i = \mathcal{M}, \mathcal{N}.$$

As the proof of Theorem 5.1 is fairly involved, I have left it to an appendix.

Before moving on, I should note that Theorem 5.1 can be strengthened slightly, to show that an ϵ-equilibrium and a δ-approximate equilibrium obtains on the entire set $[\underline{\theta}, \overline{\theta}]$, rather than on a subset with close to full measure. However, the proof of the stronger version of the theorem invokes results from the econometric theory literature less familiar to economic theorists, concerning uniform convergence in probability of stochastic functions. See the appendix for details on the alternative form of the proof.

5.2. **Affirmative Action: The Quota Game.** I now depart from the baseline color-blind model, and I derive the approximate equilibrium in the presence of race-conscious admission policies, beginning with quotas. Recall that a quota system in the finite game can be thought of as randomly selecting M prizes and setting them aside for allocation to group-$\mathcal{M}$ agents. This effectively splits the single asymmetric competition apart into two separate, symmetric competitions. As K gets large, the sample distributions of prizes reserved for each group both converge in probability to F_P. Thus, the limiting quota rule is equivalent to a set of group-specific reward functions of the form

$$\pi_i^q(b) = F_P^{-1}\left[G_i^q(b)\right], \ i = \mathcal{M}, \mathcal{N}. \tag{4}$$

Intuitively, the quantiles of the group-specific grade distributions are mapped into the corresponding quantiles of the prize distribution.

In equilibrium, the utility for a group-i student with cost θ achieving a grade of s is

$$\Pi_i^q(s, \theta) = F_P^{-1}\left[G_i^q(s)\right] - \mathcal{C}(s; \theta) = F_P^{-1}\left(1 - F_i\left[\psi_i^q(s)\right]\right) - \mathcal{C}(s; \theta).$$

This is identical to payoffs in the color-blind game, except that the unconditional cost distribution has been replaced with F_i for group $i = \mathcal{M}, \mathcal{N}$. By symmetry then, equilibrium achievement will be determined by

$$(\gamma_i^q)'(\theta) = -\frac{f_i(\theta)}{f_P\left(F_P^{-1}\left(1 - F_i(\theta)\right)\right)\mathcal{C}'(\gamma_i^q(\theta); \theta)} \tag{5}$$

and boundary condition (3).

Theorem 5.2. *Given $\rho,\ \varepsilon,\ \delta > 0$, there exists $K^* \in \mathbb{N}$, and a set $E \subset [\underline{\theta}, \overline{\theta}]$ having (Lebesgue) measure $m(E) < \rho$, such that for any $K \geq K^*$, on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$ we have the following:*

(i) *$\gamma_i^q(\theta),\ i = \mathcal{M}, \mathcal{N}$ as defined by equation (5) and boundary condition (3) generates an ε-equilibrium of the K-player quota game, and*

(ii) *$\gamma_i^q(\theta)$ is a δ-approximate equilibrium for the K-player quota game, or*

$$\|\gamma_i^q(\theta) - \gamma_i^q(\theta; K)\|_{\sup} < \delta,\ \ i = \mathcal{M}, \mathcal{N}.$$

Once again, I have left the proof of Theorem 5.2 to an appendix. As before, by using a slightly more complicated proof technique, this result can be strengthened to demonstrate that γ_i^{ap} generates an ε-equilibrium and a δ-approximate equilibrium on the entire set $[\underline{\theta}, \overline{\theta}]$, rather than on a subset with nearly full measure. See the Appendix for details.

5.3. **Affirmative Action: The Admission Preference Game.** In 1978, the Supreme Court of the United States ruled in the case of Regents of the University of California v. Bakke that explicit quotas are unconstitutional. Subsequently, American college admissions boards have been forced to seek other means by which to implement AA. These alternative implementations are sometimes referred to as "admissions preferences." An admission preference rule is modeled as a grade transformation function $\tilde{S} : \mathbb{R}_+ \to \mathbb{R}_+$, where $\tilde{S}(s)$ is increasing and $\tilde{S}(s) \geq s$. Here, prizes are matched assortatively with non-minority grades and transformed minority grades

$$\{s_{w1}, \ldots, s_{wW}, \tilde{S}(s_{m1}), \ldots, \tilde{S}(s_{mM})\}.$$

In what follows, it will be convenient to derive the equilibrium in terms of inverse equilibrium strategies. Under an admission preference, minority students are repositioned ahead of their non-minority counterparts with grades of $\tilde{S}(s)$ or less. Thus, the limiting gross payoff function for group $\mathcal{M}$ is

$$\begin{aligned} \pi_{\mathcal{M}}^{ap}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}\left(\tilde{S}(s)\right) + \mu G_{\mathcal{M}}(s)\right] \\ &= F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right] + \mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right]\right)\right] \end{aligned} \tag{6}$$

and the gross payoff function for group $\mathcal{N}$ is

$$\begin{aligned} \pi_{\mathcal{N}}^{ap}(s) &= F_P^{-1}\left[(1-\mu)G_{\mathcal{N}}(s) + \mu G_{\mathcal{M}}\left(\tilde{S}^{-1}(s)\right)\right] \\ &= F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right] + \mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(\tilde{S}^{-1}(s))\right]\right)\right]. \end{aligned} \tag{7}$$

The intuition for the above expressions is similar to the intuition for the color-blind and quota reward functions: the limiting mechanism maps the quantiles of a (mixed) distribution into the corresponding prize quantiles. For non-minorities, it is a mixture of the distributions of non-minority grades and transformed minority grades. For minorities, it is a mixture of the distributions of minority grades and *inverse-transformed* non-minority grades. Thus, a student's standing with respect to members of his own group doesn't change, but standing with respect to members of the other group does. For minorities it changes in a positive direction according to $\tilde{S}$, and for non-minorities it changes in a negative direction, since $\tilde{S}^{-1}$ lies below the 45°-line.

However, the introduction of a preference function $\tilde{S}$ introduces some complications into the analysis. Note that equation (7) only holds for s such that $\tilde{S}^{-1}(s) \geq 0$, because one can only invert grades in the range of the function $\psi^{ap}_{\mathcal{M}}$. If $\tilde{S}$ passes through the origin, this condition is satisfied for every grade in the choice set. On the other hand, there is an interesting class of admission preference rules which do not pass through the origin.[15] An example is an affine rule of the form

$$\tilde{S}(s) = \Delta_1 + \Delta_2 s,$$

where minority students receive a fixed subsidy of Δ_1, regardless of their grade. In that case, non-minorities whose grades are less than $\tilde{S}(0)$ are placed behind *all* minority students, meaning that they compete only with other non-minority students whose grades are less than $\tilde{S}(0)$. This leads to the following proposition:

Proposition 5.3. *In the college admissions game with an admission preference mechanism $\tilde{S}$, where $\tilde{S}(0) = \Delta > 0$, it follows that group-$\mathcal{N}$ players with equilibrium grades below $\tilde{S}(0)$ behave as they would under a quota rule.*

Proof: Let θ_Δ denote the non-minority type who's equilibrium grade is Δ and let

$$p_\Delta = F_P^{-1}\left(1 - [(1-\mu)F_{\mathcal{N}}(\theta_\Delta) + \mu F_{\mathcal{M}}(\theta_\Delta)]\right)$$

denote the highest prize awarded to agents whose transformed bids are Δ or less. Also, let $\mathcal{P}_\Delta = [0, p_\Delta]$. On the interval $[\theta_\Delta, \overline{\theta}]$, group $\mathcal{N}$ agents know that they are competing only among themselves for the lowest mass

$$\nu = (1-\mu)F_P(p_\Delta) = (1-\mu)\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\Delta)\right] + \mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{N}}(\Delta)\right]\right)\right]$$

of prizes since the grade markup necessarily places them behind every minority student (note that ν is also the mass of high-cost, group-$\mathcal{N}$ agents receiving prizes in $\mathcal{P}_\Delta$). It is

[15] An additive admission preference $\tilde{S}(s) = s + \Delta$ was explicitly used in undergraduate admissions at the University of Michigan. Admissions decisions were based on an index ranging from 0-120, with a bonus of 20 points being assessed to all students from underrepresented racial minority groups. This policy was in place until 2003 when the Supreme Court ruled in a joint opinion on *Gratz v. Bollinger* and *Grutter v. Bollinger*, that the bonus was unconstitutional. The opinion of the Court stated, somewhat vaguely, that while the Michigan rule was too "narrowly defined" and "mechanical," universities still have the right to consider race as a "plus factor" in admissions decisions. This abrogated a 1996 ruling to the contrary by the US 5^{th}-Circuit Court in the case of *Hopwood v. Texas*.

as if they are playing a game where the prize distribution is

$$F_{P_\Delta}(p) = \frac{F_P(p)}{\nu},\ p \in [0, F_P^{-1}(\nu)]$$

and where the distribution over competition is

$$F_{\mathcal{N}_\Delta}(\theta) = \frac{F_{\mathcal{N}}(\theta) - (1-\nu)}{\nu},\ \theta \in [\theta_\Delta, \overline{\theta}].$$

Following a similar argument as in the proof of Theorem 5.2, the limiting objective function for high-cost agents from group $\mathcal{N}$ is

$$F_{P_\Delta}^{-1}\left(1 - F_{\mathcal{N}_\Delta}\left[\psi_{\mathcal{N}}^{ap}(s)\right]\right) - \mathcal{C}(s;\theta).$$

Since $F_{P_\Delta}^{-1}(r) = F_P^{-1}(\nu r),\ r \in [0,1]$, the objective can be rewritten and rearranged as follows:

$$\begin{aligned}&F_P^{-1}\left[\nu\left(1 - \frac{F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right] - (1-\nu)}{\nu}\right)\right] - \mathcal{C}(s;\theta)\\ &= F_P^{-1}\left(1 - F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right]\right) - \mathcal{C}(s;\theta),\end{aligned}$$

which is exactly the same limiting objective function as under a quota. Since the boundary condition is also the same it follows that on the interval $[\theta_\Delta, \overline{\theta}]$, we have $\gamma_{\mathcal{N}}^{ap}(\theta) = \gamma_{\mathcal{N}}^{q}(\theta)$ and $\theta_\Delta = \psi_{\mathcal{N}}^{q}(\Delta)$ from which the result follows. This also provides a boundary condition $\gamma_{\mathcal{N}}^{ap}\left[\psi_{\mathcal{N}}^{q}(\Delta)\right] = \Delta$ for the solution of $\gamma_{\mathcal{N}}^{ap}$ on the lower interval $[\underline{\theta}, \theta_\Delta]$. ■

Knowing how $\gamma_{\mathcal{N}}^{ap}$ behaves on the upper type interval (if there is one), I have a boundary condition for non-minorities on the lower interval $[\underline{\theta}, \theta_\Delta]$ for general $\tilde{S}$. Before proceeding, it will be useful to observe that the gross payoff functions satisfy $\pi_{\mathcal{M}}^{ap}(s) = \pi_{\mathcal{N}}^{ap}(\tilde{S}(s))$.

On the lower interval, the limiting objective functions for groups $\mathcal{M}$ and $\mathcal{N}$ are, respectively,

$$F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(\tilde{S}(s))\right] + \mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(s)\right]\right)\right] - \mathcal{C}(s;\theta),\ s \geq 0$$

and

$$F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi_{\mathcal{N}}^{ap}(s)\right] + \mu F_{\mathcal{M}}\left[\psi_{\mathcal{M}}^{ap}(\tilde{S}^{-1}(s))\right]\right)\right] - \mathcal{C}(s;\theta),\ s \geq \tilde{S}(0)$$

and the FOCs for $\mathcal{M}$ and $\mathcal{N}$, respectively, are

$$-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\right](\psi^{ap}_{\mathcal{N}})'(\tilde{S}(s))\tilde{S}'(s)+\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right](\psi^{ap}_{\mathcal{M}})'(s)}{f_P\left[\Pi^{ap}_{\mathcal{M}}(s)\right]}=\mathcal{C}'(s;\theta)$$

and

$$-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right](\psi^{ap}_{\mathcal{N}})'(s)+\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(\tilde{S}^{-1}(s))\right]\frac{(\psi^{ap}_{\mathcal{M}})'(\tilde{S}^{-1}(s))}{\tilde{S}'(\tilde{S}^{-1}(s))}}{f_P\left[\Pi^{ap}_{\mathcal{N}}(s)\right]}=\mathcal{C}'(s;\theta).$$

In equilibrium, it will be true that $\psi_i^{ap}(s)=\theta$ for group i, so by substituting and rearranging I get

$$\begin{aligned}(\psi^{ap}_{\mathcal{M}})'(s)=&-\frac{\mathcal{C}'\left[s;\psi^{ap}_{\mathcal{M}}(s)\right]f_P\left[\Pi^{ap}_{\mathcal{M}}(s)\right]}{\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right]}\\&-\frac{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\right]}{\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(s)\right]}(\psi^{ap}_{\mathcal{N}})'(\tilde{S}(s))\tilde{S}'(s)\end{aligned}\tag{8}$$

and

$$\begin{aligned}(\psi^{ap}_{\mathcal{N}})'(s)\tilde{S}'(\tilde{S}^{-1}(s))=&-\frac{\mathcal{C}'\left[s;\psi^{ap}_{\mathcal{N}}(s)\right]f_P\left[\Pi^{ap}_{\mathcal{N}}(s)\right]\tilde{S}'(\tilde{S}^{-1}(s))}{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right]}\\&-\frac{\mu f_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{M}}(\tilde{S}^{-1}(s))\right]}{(1-\mu)f_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right]}(\psi^{ap}_{\mathcal{M}})'(\tilde{S}^{-1}(s)).\end{aligned}\tag{9}$$

Equations (8) and (9), with boundary conditions complete the solution for $(\gamma^{ap}_{\mathcal{M}},\gamma^{ap}_{\mathcal{N}})$. By evaluating equation (9) at $\tilde{S}(s)$ and substituting it into the FOC for minorities, equation (8) reduces to

$$\mathcal{C}'\left[s;\psi^{ap}_{\mathcal{M}}(s)\right]=\mathcal{C}'\left[\tilde{S}(s);\psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\right]\tilde{S}'(s),\tag{10}$$

which provides a relation between grade selection in the two groups. As it turns out, equation (10) is important for characterizing the effects of $\tilde{S}$ on minority bidding. The solution for equilibrium grades under a general admission preference $\tilde{S}$ is given by Proposition (5.3); equations (9) and (10) and boundary condition $\psi^{ap}_{\mathcal{N}}(\Delta)=\psi^{q}_{\mathcal{N}}(\Delta)=\theta_\Delta$, where $\Delta=\tilde{S}(0)$. Of course, the following theorem is needed to validate this claim.

Theorem 5.4. *In the college admission game with an admission preference $\tilde{S}$ satisfying assumptions 3.6 and 3.7, given $\rho,\ \varepsilon,\ \delta>0$, there exists $K^*\in\mathbb{N}$, and a set $E\subset[\underline{\theta},\overline{\theta}]$ having (Lebesgue) measure $m(E)<\rho$, such that for any $K\geq K^*$, on any closed subset of $[\underline{\theta},\overline{\theta}]\setminus E$ we have the following:*

(*i*) *an ε-equilibrium of the K-player admission preference game is generated by $\gamma_i^{ap}(\theta),\ i=\mathcal{M},\mathcal{N}$ as defined by Proposition (5.3), equation (9), equation (10), boundary condition*

(3) for non-minorities and boundary condition

$$\mathcal{C}'\left[0;\theta^*\right] = \mathcal{C}'\left[\Delta;\theta_\Delta\right]\tilde{S}'(0)$$

for minorities, where $\theta^* = \inf\left\{\theta \;:\; \gamma^{ap}_{\mathcal{M}}(\theta) = 0\right\}$, $\theta_\Delta = \psi^{ap}_{\mathcal{N}}(\Delta)$, *and* $\Delta = \tilde{S}(0)$; *and*

(ii) $\gamma_i^{ap}(\theta)$ *is a* δ*-approximate equilibrium for the K-player quota game, or*

$$\|\gamma_i^{ap}(\theta) - \gamma_i^{ap}(\theta;K)\|_{\sup} < \delta, \;\; i = \mathcal{M},\mathcal{N}.$$

Proof: The proof is similar to that for Theorem 5.1. ■

As before, by using a more complicated proof technique, this result can be strengthened to demonstrate that γ_i^{ap} generates an ε-equilibrium and a δ-approximate equilibrium on the entire set $[\underline{\theta},\overline{\theta}]$, rather than on a subset with nearly full measure. See the Appendix for details.

6. SPECIAL CASE: UNIFORM PRIZES AND LINEAR COSTS

As the above discussion demonstrates, the present model of market competition is flexible enough to handle a wide range of specifications. In this section I shall impose two simplifications in order to facilitate qualitative characterizations of model equilibria. First, since the objective of this research is to characterize the effects of policy changes, I shall abstract away from the intricacies of the "supply side" of the market, and assume that prizes (*e.g.*, college seats) are distributed uniformly on the unit interval. Second, in order to simplify the analysis I henceforth adopt a cost function that is linear in achievement:

$$\mathcal{C}(s;\theta) = \theta s.$$

In order to derive qualitative comparisons of behavioral responses for different groups that are unevenly affected by AA, it will be helpful to assume that the type distributions are ordered by likelihood ratio dominance, or

Assumption 6.1. $h(\theta) = \frac{f_{\mathcal{M}}(\theta)}{f_{\mathcal{N}}(\theta)}$ is a strictly increasing function on $[\underline{\theta},\overline{\theta}]$.

Likelihood ratio dominance is essentially a strong form of first-order stochastic dominance.[16] In other words, the game is assumed to be asymmetric in the sense that minority study costs are higher on average.

The asymmetry assumption is not intended to imply that there are fundamental differences in inherent ability across the two groups, as types reflect a myriad of environmental factors as well. Rather, it is in keeping with arguments made by proponents of AA regarding systemic competitive disadvantages for minorities, due to various historical factors. For example, White children in the United States, on average, are more

[16] An excellent exposition of this topic can be found in Krishna [15].

affluent and attend primary and secondary schools that are better funded than African-Americans. The idea behind cost asymmetry is that an average minority student must expend more personal effort to overcome the environmental obstacles—poverty, poor health-care, lower quality K-12 education, *etc.*—eroding his competitive edge.[17]

6.1. **Full Quota.** Some interesting observations can be made about how behavior changes when moving from a color-blind policy to a quota. It turns out that with quota admissions, the highest performing minority students decrease their academic achievement and the lowest performing students increase it. For non-minorities the change is exactly the opposite: high performers increase their effort and low performers decrease it. This result is formalized in the following theorem.

Theorem 6.2. *If prizes are uniform, $\mathcal{C}(s;\theta) = \theta s$, and $F_{\mathcal{M}}(\theta)$ dominates $F_{\mathcal{N}}(\theta)$ according to the likelihood ratio order, then there exists $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that*

(i) for minority competitors, $\gamma^q_{\mathcal{M}}(\theta) < (>) \gamma^{cb}(\theta)$ for each $\theta < (>) \theta^*$; and

(ii) for non-minority competitors, $\gamma^q_{\mathcal{N}}(\theta) > (<) \gamma^{cb}(\theta)$ for each $\theta < (>) \theta^*$.

Proof: First note that with uniform prizes and linear costs, equations (2) and (5) simplify to

$$\begin{aligned} \gamma^{cb}(\theta) &= \int_{\theta}^{\overline{\theta}} \frac{\mu f_{\mathcal{M}}(u) + (1-\mu) f_{\mathcal{N}}(u)}{u} du \quad \text{and} \\ \gamma^q_i(\theta) &= \int_{\theta}^{\overline{\theta}} \frac{f_i(u)}{u} du, \quad i \in \{\mathcal{M}, \mathcal{N}\}, \end{aligned} \tag{11}$$

from which it can easily be seen that $\gamma^{cb}(\theta) = \mu \gamma^q_{\mathcal{M}}(\theta) + (1-\mu)\gamma^q_{\mathcal{N}}(\theta)$.

By likelihood ratio dominance, it follows that $f_{\mathcal{M}}$ and $f_{\mathcal{N}}$ have a single crossing at some $\tilde{\theta} \in (\underline{\theta}, \overline{\theta})$, and $f_{\mathcal{M}}(\theta) > f_{\mathcal{N}}(\theta)$ for each $\theta > \tilde{\theta}$. From this and equations (11) it follows that $\gamma^q_{\mathcal{M}}(\theta) > \gamma^q_{\mathcal{N}}(\theta)$ for each $\theta \in [\tilde{\theta}, \overline{\theta})$. Moreover, if the functions cross at some θ^* then it must be on the interval $[\underline{\theta}, \tilde{\theta})$. Note that

$$\gamma^q_{\mathcal{M}}(\underline{\theta}) = \int_{\underline{\theta}}^{\overline{\theta}} f_{\mathcal{M}}(u) \frac{1}{u} du < \int_{\underline{\theta}}^{\overline{\theta}} f_{\mathcal{N}}(u) \frac{1}{u} du = \gamma^q_{\mathcal{N}}(\underline{\theta}),$$

where the inequality follows from the fact that each side is an expectation and $f_{\mathcal{N}}$ places more weight on larger values of the function $\frac{1}{u}$ (*i.e.*, smaller values of u) on the interval

[17] There is some empirical evidence consistent with this view. Neal and Johnson [16] find that for the Armed Forces Qualification Test, "family background variables that affect the cost or difficulty parents face in investing in their children's skill explain roughly one third of the racial test score differential" (pg. 871). Fryer and Levitt [8] analyze data on racial test-score gaps among elementary school children in an attempt to uncover the causes. They find that by controlling for socioeconomic status and other environmental factors which vary substantially by race, test-score gaps significantly decrease, but not entirely. They test various hypotheses to explain the remainder of the gap, and find that disparities in school quality is the only one not rejected by the data.

$[\underline{\theta}, \overline{\theta}]$. Then by likelihood ratio dominance and continuity it follows that θ^* exists on the open interval $(\underline{\theta}, \tilde{\theta})$ and is unique. Finally, the result of the theorem follows from the fact that γ^{cb} is a convex combination of $\gamma^q_{\mathcal{M}}$ and $\gamma^q_{\mathcal{N}}$. ■

Theorem 6.2 highlights some interesting facets of a student's decision problem. The intuition stems from the fact that a quota mechanism splits the competition apart into two separate competitions. In doing so, it alters the distribution of competition that each minority student faces from $(1-\mu)F_{\mathcal{N}}(\theta) + \mu F_{\mathcal{M}}(\theta)$ to $F_{\mathcal{M}}(\theta)$. For a low-performing (*i.e.*, high-cost) student, the mass of competitors at an advantage to him is relatively large before the change. Since costs are sunk, it is not worthwhile for such a student to exert much effort when competing with the population at large. This phenomenon is commonly known in the all-pay auctions literature as the *discouragement effect*. However, if the student faces only minority competitors (with higher costs on average), the discouragement effect is mitigated and effort increases.

For high-performing (*i.e.*, low-cost) minorities, the effect is reversed: when the relative mass of similar low-cost opponents decreases there is less need to aggressively outperform the competition in order to win a highly valued seat. Thus, effort decreases among top minority students. The effects for non-minority students in each category are exactly the opposite, for the same intuition. The discouragement effect for high-cost non-minorities is exacerbated by moving to a quota system, and low-cost non-minorities must compete more aggressively against a set of competitors whose costs are on average lower.

An evaluation of a quota in terms of the policy objectives outlined in Section 3.3, as compared to the baseline color-blind mechanism, is provided in the following corollary:

Corollary 6.3. *Maintain the assumptions of Theorem 6.2 and let $\theta^* \in (\underline{\theta}, \overline{\theta})$ be the cutoff type defined there. Then the following statements immediately follow:*

(i) (minority achievement) $G^q_{\mathcal{M}}(s) > (<) G^{cb}_{\mathcal{M}}(s)$ for each $s > (<) \gamma^{cb}(\theta^*)$,

(ii) (non-minority achievement) $G^q_{\mathcal{N}}(s) < (>) G^{cb}_{\mathcal{N}}(s)$ for each $s > (<) \gamma^{cb}(\theta^*)$

(iii) (achievement gap) $\mathcal{A}^q(q) > (<) \mathcal{A}^{cb}(q)$ for each $q > (<) \mu F_{\mathcal{M}}(\theta^*) + (1-\mu) F_{\mathcal{N}}(\theta^*)$, and

(iv) (enrollment gap) $\mathcal{E}^q(q) < \mathcal{E}^{cb}(q)$ for all $q \in (0,1)$.

In words, the effect on the achievement gap is mixed because of how the different groups respond to the policy. A quota widens it among the best and brightest students, since low-cost minorities decrease achievement and low-cost non-minorities increase achievement, relative to the color-blind case. At the bottom end of the score distribution we see a narrowing of the achievement gap, as high-cost minorities work harder and their non-minority counterparts decrease output. Finally, an attractive quality of the quota system is that by design it achieves an enrollment gap of zero (this is true in the general case as well). Thus, when types are ordered by stochastic dominance, a quota is guaranteed to produce an improvement over a color-blind mechanism in terms of the enrollment gap.

6.2. **Admission Preferences.** When costs are linear, equation (10) reduces to

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\tilde{S}'(s). \tag{12}$$

As mentioned previously, this equation reveals much about minority grade selection under preference rules. For beginners, it indicates under what circumstances the admission preference rule will lead to a mass-point of students achieving a grade of zero, as outlined in the following proposition.

Theorem 6.4. *In the college admissions game, assume that study costs are of the form $C(s;\theta) = \theta s$ and assume an admission preference mechanism $\tilde{S}$ satisfying assumptions 3.6 and 3.7. Moreover, define $\Delta \equiv \tilde{S}(0)$ and $\theta_\Delta \equiv \psi^{ap}_{\mathcal{N}}(\Delta) = \psi^{q}_{\mathcal{N}}(\Delta)$. Then the following results follow:*

(i) If $\tilde{S}'(0) \geq (>) \frac{\overline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)}$ the grade achieved by a minority student with the highest possible cost type is non-negative (strictly positive).

(ii) If $\frac{\underline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)} < \tilde{S}'(0) < \frac{\overline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)}$ there is a positive mass $\zeta \in (0,1)$ of minority students who choose equilibrium grades of zero.

(iii) If $\tilde{S}'(0) < \frac{\underline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)}$ all minority students choose equilibrium grades of zero.

Proof: From equation (12) it follows that

$$\inf \psi_{\mathcal{M}}(0) = \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0),$$

which solves for the lowest minority type who achieves a grade of zero. Statements *(i)*, *(ii)* and *(iii)* then follow from substituting the left-hand side to test whether

$$\overline{\theta} \gtreqless \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0)$$

and

$$\underline{\theta} \gtreqless \psi_{\mathcal{N}}(\Delta)\tilde{S}'(0). \quad \blacksquare$$

Note that Theorem 6.4 holds for general prize distributions, and does not depend on stochastic ordering of types.

At this point I will further simplify the analysis by focusing on an additive admission preference of the form

$$\tilde{S}(s) = s + \Delta.$$

Aside from being a useful illustrative tool, this policy is of particular interest as it was previously used in undergraduate admissions at the University of Michigan. For that reason I shall henceforth refer to it as the "Michigan rule." Theorem 6.4 shows that for any level of Δ, a Michigan rule will lead to a mass point of minority students with zero achievement in equilibrium. This is because it has a slope of 1, whereas $\frac{\overline{\theta}}{\psi^{ap}_{\mathcal{N}}(\Delta)} > 1$ for any positive Δ. However, if one were to consider a more general affine admission preference, say

$$\tilde{S}(s) = \Delta_0 + \Delta_1 s$$

then as Proposition 6.4 shows, the slope coefficient could be chosen so as to eliminate the mass point of minority students achieving grades of zero.[18]

I shall now proceed to derive the equilibrium under a Michigan rule. In this case, equation (10) further reduces to

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(s + \Delta), \tag{13}$$

which indicates that a minority student with type θ will achieve a grade of exactly Δ less than his non-minority counterpart with the same type. Recall that non-minority students whose grades are less than Δ will behave the same as under a quota rule, giving a boundary condition of

$$\psi^{ap}_{\mathcal{N}}(\Delta) = \psi^{q}_{\mathcal{N}}(\Delta) = \theta_{\Delta}, \tag{14}$$

and minority students with costs above θ_{Δ} choose a grade of zero.

Equation (13) can be substituted back into the decision problem of $\mathcal{N}$ agents to get

$$F_P^{-1}\left[1 - \left((1-\mu)F_{\mathcal{N}}\left[\psi^{ap}_{\mathcal{N}}(s)\right] + \mu F_{\mathcal{M}}\left[\psi^{ap}_{\mathcal{N}}(s)\right]\right)\right] - \theta s,$$

which gives the familiar FOC:

$$(\gamma^{ap}_{\mathcal{N}})'(\theta) = -\frac{(1-\mu)f_{\mathcal{N}}(\theta) + \mu f_{\mathcal{M}}(\theta)}{f_P\left[F_P^{-1}\left(1 - [(1-\mu)F_{\mathcal{N}}(\theta) + \mu F_{\mathcal{M}}(\theta)]\right)\right]\theta}. \tag{15}$$

Recall that this is the same as the differential equation arising from the FOC under a color-blind rule. The important difference here is that its solution depends on a different boundary condition, given by equation (14).

An evaluation of a Michigan rule in terms of the policy objectives outlined in Section 3.3, as compared to the baseline color-blind mechanism, is provided in the following theorem:

[18]Chapter 3 of this thesis considers an affine admission preference in depth. As it turns out, the AA policy recovered from the US college admissions data is affine with both a positive slope and a positive intercept.

Theorem 6.5. *Let prizes be uniform, let achievement costs be of the form $\mathcal{C}(s;\theta) = \theta s$, let $F_{\mathcal{M}}(\theta)$ dominate $F_{\mathcal{N}}(\theta)$ according to the likelihood ratio order, and assume a Michigan admission preference $\widetilde{S}$ with fixed grade markup $\Delta < \gamma^q_{\mathcal{N}}(\theta^*)$, where θ^* is the cutoff type in Proposition 6.2. Then the following statements are true:*

- *(i) (minority achievement) $G^{ap}_{\mathcal{M}}(s) > G^{cb}_{\mathcal{M}}(s)$ for all s,*
- *(ii) (non-minority achievement) $G^{ap}_{\mathcal{N}}(s) < G^{cb}_{\mathcal{N}}(s)$ for all s*
- *(iii) (achievement gap) $\mathcal{A}^{ap}(q) > \mathcal{A}^{cb}(q)$ for all q, and*
- *(iv) (enrollment gap) $\mathcal{E}^q(q) < (=)\mathcal{E}^{cb}(q)$ for all $q < (>)\mu F_{\mathcal{M}}(\theta_\Delta) + (1-\mu)F_{\mathcal{N}}(\theta_\Delta)$, where $\theta_\Delta = \psi^{ap}_{\mathcal{N}}(\Delta)$.*

Proof: The fact that non-minorities decrease their achievement relative to the color-blind case follows immediately from the above derivations. First, recall that $\gamma^{ap}_{\mathcal{N}}(\theta) = \gamma^q_{\mathcal{N}}(\theta) < \gamma^{cb}_{\mathcal{N}}(\theta)$ on the interval $[\theta_\Delta, \overline{\theta}]$ (see Theorem 5.3). Second, note that equations (15) and (14) imply that non-minority achievement parallels color-blind achievement, but from a lower initial condition. Note also that by Theorem 6.4 and equation (13) minority types below θ_Δ reduce achievement by exactly Δ more than their non-minority counterparts, and all other non-minority types achieve a grade of zero. From this it follows that the achievement gap is unambiguously widened at all quantiles of the grade distribution. Finally, by the same argument, the enrollment gap is unaffected for types above θ_Δ because in equilibrium a Michigan rule merely compensates for the minority behavioral response and does not alter relative standing between the two groups. ■

This result is significant for several reasons. First, it highlights an important aspect of the behavioral response to admission preferences. Although the policy-maker may intend to bolster minority students' competitive edge with a grade subsidy, a rational student may simply treat the markup as a direct utility transfer and reduce achievement. When costs are linear and the markup does not depend on output, minority students scale back achievement by exactly the amount of the markup, relative to non-minority counterparts.[19] This picture becomes more bleak when one recognizes that non-minorities also reduce their achievement, relative to the color-blind case. In other words, when moving from color-blind allocations to a Michigan rule, a markup of Δ leads to a grade reduction of *more* than Δ for *all* minority students, and a smaller reduction for all non-minorities as well. The implication is that a Michigan rule is unambiguously detrimental to effort incentives.

[19]When costs are convex in achievement, the effect is less straightforward (see equation (10)) and complete consumption of the grade boost will not obtain in general. However, it will generally be the case that the beneficiaries of an admission preference will use at least some portion of the grade boost as a direct utility subsidy, rather than using it only to bolster their competitive edge.

Second, given the above facts about the relation between minority and non-minority achievement under a Michigan rule, it immediately follows that the policy will lead to a widening of the racial achievement gap at every point in the type support, relative to a color-blind policy. This is because like types have the same achievement under a color-blind rule, whereas the difference in achievement between a minority and a non-minority with type θ under the Michigan rule is $\min\{\Delta, \gamma^{ap}_{\mathcal{M}}(\theta)\}$, which is strictly positive for any $\theta < \overline{\theta}$.

Finally, these facts also imply that an admission preference rule is an ineffective means for helping minorities into better colleges. With linear costs, students choose grades so that once the fixed markup is assessed relative standings between minorities and non-minorities is the same as under a color-blind allocation mechanism. The only change that occurs is among minorities scoring zero and non-minorities scoring below Δ. Thus, a Michigan rule has *no allocational effect* except to re-shuffle allocations in the lower tail of the prize distribution. Enrollment at the most selective schools will be identical to enrollment under a color-blind admission policy.

The intuition behind the behavioral response to an admission preference stems from equation (12), reproduced below for convenience:

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(\tilde{S}(s))\tilde{S}'(s).^{20}$$

Minority students in a color-blind world compete on the margin with non-minority counterparts of roughly the same cost type, but a grade markup alters this relationship. In general, students respond to the policy by adjusting behavior until once again they are competing on the margin with non-minority counterparts on the same competitive standing. As the above example illustrates, behavioral responses to the policy may be undesirable, and they can even nullify the ability of the policy to alter allocations by changing the relative standing between minorities and non-minorities. Expression (12) leads to a necessary condition for the minority behavioral response to be in a desirable direction.

Theorem 6.6. *Let achievement costs be of the form $C(s;\theta) = \theta s$, let the admission preference policy $\widetilde{S}$ satisfy assumptions 3.6 and 3.7, and define $\widetilde{M}(s) \equiv \widetilde{S}(s) - s$ to be the score markup implied by the admission preference. Then for $\theta \in [\underline{\theta}, \overline{\theta}]$, we have*

$$\mathcal{A}^{ap}\left[F_{\mathcal{M}}(\theta)\right] \leq \mathcal{A}^{cb}\left[F_{\mathcal{M}}(\theta)\right]$$

[20] Recall that this expression results in part from the linear functional form of costs assumed in this section. I concentrate on the restricted version here for expositional purposes: it vastly simplifies the intuition. For the original expression which incorporates general forms of utility curvature, see equation (10).

only if the marginal markup for minority type θ is positive, or

$$\widetilde{M}'\left(\gamma^{ap}_{\mathcal{M}}(\theta)\right) > 0.$$

Proof: The result of the theorem follows straightforwardly from two facts. The first is that $\widetilde{M}'(s) > 0 \iff \widetilde{S}'(s) > 1$. The second fact is that $\mathcal{A}^{ap}\left[F_{\mathcal{M}}(\theta)\right] \leq \mathcal{A}^{cb}\left[F_{\mathcal{M}}(\theta)\right]$ only if $\gamma^{ap}_{\mathcal{M}}(\theta) \geq \gamma^{ap}_{\mathcal{N}}(\theta)$. This is because $\gamma^{cb}_{\mathcal{M}}(\theta) = \gamma^{cb}_{\mathcal{N}}(\theta)$ and all of the achievement gap in the color-blind game comes from asymmetry in the type distributions. Thus, in order to close the gap it is necessary for minority students to achieve higher scores than their non-minority counterparts of the same type.

Fix $\theta \in [\underline{\theta}, \overline{\theta}]$ and let $s = \gamma^{ap}_{\mathcal{M}}(\theta)$. If $\widetilde{S}'(s) = 1$, then (12) reduces to

$$\psi^{ap}_{\mathcal{M}}(s) = \psi^{ap}_{\mathcal{N}}(\tilde{S}(s)),$$

from which it follows that $\gamma^{ap}_{\mathcal{N}}(\theta) - \gamma^{ap}_{\mathcal{M}}(\theta) = \widetilde{M}(s) > 0$. By monotonicity, a similar inequality holds when $\widetilde{S}'(s) < 1$. ■

Note that Theorem 6.6 holds for general prize distributions and does not require assumptions about stochastic ordering of types. The proof of the theorem highlights a tension between the level of the admission preference and it's slope. As equation (12) illustrates, an increase in the level of the markup (holding slope fixed) causes minorities to reduce output, relative to their non-minority counterparts. On the other hand, an increase in the marginal markup (holding the level of the markup fixed at a given point) has the opposite effect. The critical flaw in a Michigan rule is that its slope is identical to a color-blind markup $\widetilde{S}^{cb}(s) = s$, and it simply offers a higher level of assistance, Δ.

Another way to think about this is that a Michigan rule fails to reward achievement: with a zero marginal markup, students achieving higher scores get the same boost as everyone else. In essence, Theorem 6.6 shows that in order for AA to be effective, the assistance it renders to minority students must be merit based. A multiplicative rule of the form $\widetilde{S}(s) = (1+r)s,\ r > 0$ is an example of an admission preference with a positive marginal markup, $\widetilde{M}'(s) = r$. As general results other than those above are difficult to prove, I shall defer discussion of a multiplicative preference to the following section, where I illustrate the model using numerical methods.

6.3. **Example: Pareto Types.** In this section I illustrate the model by computing approximate equilibria for a simple special case. Prizes are distributed uniformly on the interval $[0, 100]$, so that $F_P(p) = \frac{p}{100}$. The fraction of minority college candidates is $\mu = 0.25$ and types follow a Pareto distribution, with the upper tail truncated to the interval $[1, 5]$. For group i the type distribution is

$$F_i(\theta) = \frac{1 - \theta^{-\kappa_i}}{1 - \overline{\theta}^{-\kappa_i}},\ \kappa_i > 0,\ i = \mathcal{M}, \mathcal{N},$$

where $\kappa_{\mathcal{M}} = 0.1$ and $\kappa_{\mathcal{N}} = 1.5$, so that stochastic dominance holds. The further $\kappa_{\mathcal{M}}$ is from $\kappa_{\mathcal{N}}$, the more pronounced is the asymmetry across groups. All parameters in this section were chosen for purely illustrative purposes, with the exception of Δ, which is discussed below. With the above parameters specified, solving for the equilibrium is a simple mater of integrating differential equations.

A value of Δ was chosen based on the above parameters to facilitate comparisons between a quota and a Michigan rule. A key characteristic of a quota is that it ensures that the average prize value allocated to members of each group is the same. Thus, I compute the fixed grade subsidy Δ^* that equates the average prize value awarded to each group. This provides some basis for comparison of the two different policies, as they are both designed to achieve a common objective.[21]

Computing Δ^* under uniformly-distributed prizes is fairly simple. Recall from Section 6.2 that when costs are linear, an additive grade subsidy only alters equilibrium allocations among agents who grade less than Δ. Once again, let θ_Δ denote the player type that submits a grade of Δ for group $\mathcal{N}$ and let $p_\Delta = F_P^{-1}\left[1 - \{(1-\mu)F_{\mathcal{N}}(\theta_\Delta) + \mu F_{\mathcal{M}}(\theta_\Delta)\}\right]$ denote the top prize allocated to players whose transformed grades are Δ or less.

Within the interval $[0, p_\Delta]$, the top mass μ of prizes are awarded to group $\mathcal{M}$ and the rest are given to group $\mathcal{N}$. Thus, the average prize given to candidates with costs $c \leq \theta_\Delta$ in group $\mathcal{M}$ and $\mathcal{N}$ are, respectively, $(p_\Delta + \mu p_\Delta)/2$ and $\mu p_\Delta/2$. The average prize awarded to players of either group with types above θ_Δ are the same–recall that transformed equilibrium grades are the same for a given θ in this interval–and are given by $(\overline{p} + p_\Delta)/2$. Therefore, the average prize allocated to group $\mathcal{M}$ candidates is

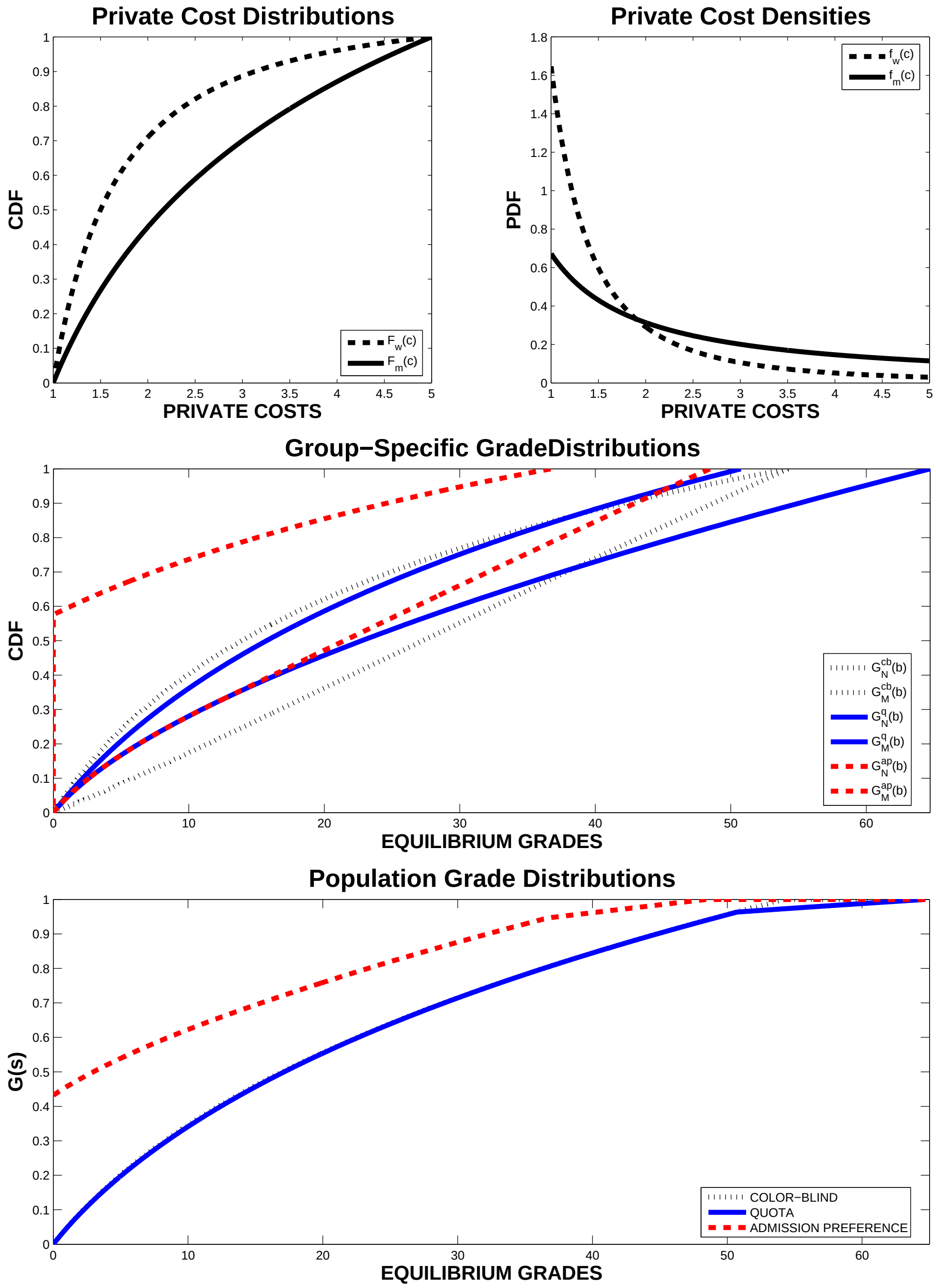
$$[1 - F_{\mathcal{M}}(\theta_\Delta)]\,\frac{p_\Delta + \mu p_\Delta}{2} + F_{\mathcal{M}}(\theta_\Delta)\frac{\overline{p} + p_\Delta}{2} \tag{16}$$

and the average prize for group $\mathcal{N}$ is

$$[1 - F_{\mathcal{N}}(\theta_\Delta)]\,\frac{\mu p_\Delta}{2} + F_{\mathcal{N}}(\theta_\Delta)\frac{\overline{p} + p_\Delta}{2}. \tag{17}$$

[21]The defining characteristic of a quota is that it equates all moments of the group allocation distributions, but this is impossible to do with a Michigan rule in an asymmetric game. Here I focus on the first central moment for illustrative purposes.

FIGURE 1. Numerical Example: Pareto Distributed Types



Thus, Δ^* is determined by the following equality

$$[1 - F_{\mathcal{M}}(\theta_{\Delta^*})]\,\frac{p_{\Delta^*} + \mu p_{\Delta^*}}{2} + F_{\mathcal{M}}(\theta_{\Delta^*})\frac{\overline{p} + p_{\Delta^*}}{2} = [1 - F_{\mathcal{N}}(\theta_{\Delta^*})]\,\frac{\mu p_{\Delta^*}}{2} + F_{\mathcal{N}}(\theta_{\Delta^*})\frac{\overline{p} + p_{\Delta^*}}{2}. \tag{18}$$

For the above parameters, Δ^* is about 24% of the maximal grade achieved by group $\mathcal{M}$. Of course, the size of Δ^* depends on the degree of asymmetry between the two groups, and is therefore an empirical question. For example, a $(\kappa_{\mathcal{M}}, \kappa_{\mathcal{N}})$ pair of $(0.6, 1)$ cuts Δ^* roughly by half. This example merely demonstrates that in order for a Michigan rule to deliver the same average allocative effect as a quota, the fixed markup potentially must be quite large.

Figure 1 plots several objects of interest. The top two panes are the distributions and densities of types. The middle pane displays group-specific grade distributions under each of the three admission policies. A color-blind rule is denoted by a dotted line, a Michigan rule is denoted by a dashed line, and a solid line denotes a quota. For pairs of lines with the same style, the one lying to the left is the grade distribution for minorities. The bottom pane displays population grade distributions, following a similar convention. When comparing two grade distributions, keep in mind that if distribution i lies to the right of distribution j in some region, it indicates an interval of students who are enticed to achieve a higher academic output under policy i.

The middle plot gives an idea of how within-group behavior changes, and also how the achievement gap changes under different policies. As Propositions 6.4 and 6.5 suggest, the picture for an additive markup is dismal. With any Michigan rule, the policy-maker must settle for a mass-point of zero achievement in order to equalize average outcomes for each group. As this example shows, the mass point can be potentially large. The general insight here is that as asymmetry increases, a Michigan rule becomes increasingly inadequate as a policy instrument. Notice also the substantial leftward shift in non-minority grades associated with the quota-comparable Michigan rule. Recall also that allocations in the upper tail of the prize distribution are unaffected, even with this extreme version of the policy. A more definitive analysis is ultimately an empirical exercise, but this example demonstrates how an ill-designed admission preference can lead to a substantial social loss, while producing little in the way of desired change.

As outlined in Proposition 6.2, a quota rule produces some interesting benefits, relative to a color-blind system. The middle pane shows that all minorities below the 80^{th} grade percentile and all non-minorities above the 70^{th} grade percentile increase their performance, relative to a color-blind system. Of course, there are also costs involved, as all other students decrease academic output. Although it is difficult to tell from the lower pane, it turns out that a quota produces a slight first-order dominance shift in the overall population grade distribution.

In contrast, the difference from Michigan-rule grades for both color-blind and quota admissions is quite stark. The results here highlight some inaccuracies in statements made by American policy-makers that, "at their core, the Michigan policies amount to a quota system" (see Bush [19]). In fact, the contrast between the two versions of AA could not be more stark: a Michigan rule in no way resembles a quota system either in terms of its behavioral or allocative effects.

7. DISCUSSION AND CONCLUSION

In this chapter, I have explored the qualitative implications of different AA policies in college admissions. Design of the AA implementation can have significant effects on both effort choice and college placement. On the one hand, it appears that the critics of AA are correct in assuming that a tradeoff exists between equality and academic performance incentives, although the exact nature of the tradeoff—whether it results in a socially desirable change—cannot be resolved theoretically. On the other hand, proponents of AA are also correct in assuming that race-conscious admissions can potentially increase academic performance for some minorities by diminishing discouragement effects. However, in the process of leveling the playing field for some minority students, the situation is made worse for disadvantaged (*i.e.*, high-cost) non-minorities. Moreover, advantaged minorities find diminished incentives for academic performance in an environment where the competition they face is less fierce.

Although the model produces many useful qualitative insights into the college admissions problem, recovering the exact nature of the equity–achievement tradeoff for the purpose of enlightening policy decisions cannot be done without a meaningful empirical analysis. This is for two reasons. First, a comparison between the broad class of admission preference mechanisms and either a color-blind or a quota mechanism cannot be theoretically resolved within the confines of the Wilson Doctrine. This is because such an undertaking would require specific knowledge of agents' beliefs about the competition they face. Second, a meaningful policy analysis requires empirical measurement of actual AA practices. In the following chapter I structurally estimate the model in order to produce quantitative comparisons between different college admissions practices.

If we take the issues of achievement and enrollment gaps separately, somewhat more can be said qualitatively. For example, the results proven here provide a possible theoretical explanation for some well-known empirical results on the predictive power of college entrance test scores. Vars and Bowen [22] use data on SAT scores and subsequent academic performance at "highly selective" post-secondary institutions to investigate whether the entrance test score predicts college success equally well for different races. Their results indicate that

> "while SAT scores are related to college GPA for both blacks and whites, the relationship is weaker for blacks. More important, and more disturbing, at every level of SAT score, blacks earn lower grades than their white counterparts... Most sobering of all, the performance gap is greatest for the

> black students with the highest SATs. The reasons for this gap are not well understood; nevertheless, we believe that many gifted African-American students at academically selective institutions are not realizing their full academic potential."[22]

Propositions 6.2 and 6.5 may provide an explanation for this puzzle: they indicate that any type of AA based only on race (*i.e.*, both quotas and admission preferences) widens the achievement gap among the highest-performing students. This is illustrated in Figure 1, where the upper tails of the grade distributions for each group are further apart under AA than under color-blind admissions. Since these high-performing students are typically the ones who gain admission to selective institutions, it is plausible that the predictive disparity is the result of a behavioral response to AA. As illustrated in Figure 1, under a quota, achievement by top minority students is compressed to a tighter interval, relative to color-blind admissions, whereas the opposite is true for non-minority scores. For high-ability individuals, AA creates effort disincentives for minorities, relative to non-minorities, and it may also diminish separation among minorities while increasing separation among non-minorities.

Thus, to the extent that SAT scores are a meaningful measure of the human capital high-school students bring with them to college, and to the extent that the subsequent labor market resembles the competitive setting outlined in Section 3, an explanation for Vars and Bowen [22] may be that AA practices in the skilled labor market do not provide adequate incentives for top minority students to perform at their "full academic potential."

As for AA and enrollment, by construction quotas achieve 100% equal allocations in the sense that the racial makeup of student bodies at schools of all quality levels will be reflective of population proportions. On the other hand, the effectiveness of admission preferences in rearranging allocations is hampered by the rational behavioral response to this type of policy. Clearly, a policy-maker should *not* treat behavior as fixed when predicting equilibrium allotments of college seats to different groups under different policies. Indeed, in the case of a Michigan-type additive admission preference, such an assumption may result in a near total nullification of any intended change.

There are two other interesting directions for further research. First, an important related question would be how AA might affect educational attainment decisions among minorities. The current model focuses on student behavior, conditional on participation in the college market, but there is another interesting group of individuals to consider as well: those whose college/work-force decisions may be affected by a given policy.

[22]Bowen and Bok [3, Ch. 3, pp. 72–78] report similar findings on the relation between SAT scores and subsequent college GPA.

This question could be addressed by formalizing the "supply-side," being comprised of potential colleges and firms who may enter the market and supply post-secondary education services or unskilled jobs. Such a model might illuminate how the marginal agent (*i.e.*, the individual indifferent between attending college and entering the workforce) is affected by a given college admission policy. This would help to characterize the effect of Affirmative Action on the total mass of minorities enrolling in college.

Finally, the eventual goal for this line of research should be to answer the question of how AA helps or hinders the objective of erasing the residual effects of past institutionalized racism. This will require a more general dynamic model in which the policy-maker is not only concerned with short-term outcomes for students whose types are fixed; but also with the long-run evolution of the type distributions. Empirical evidence suggests that academic competitiveness is determined by things such as affluence and parents' education. If AA affects performance and outcomes for current minority students in a certain way, the next question is what effect it might that have on their children's competitiveness when the next generation enters high school. If a given policy produces the effect of better minority enrollment and higher achievement in the short-run then one might conjecture that a positive long-run effect will be produced. However, given the mixed picture on the various policies considered in this paper, it seems evident that a long-run model is needed in order to give meaningful direction to forward-looking policy-makers. The theory developed here will hopefully serve as a basis for answering these important questions in the future.

APPENDIX

A.0.1. **Proof of Proposition 3.8. Proof:** Existence and monotonicity is a straightforward application of Athey [2, Theorem 3] who proves existence and monotonicity of a pure-strategy equilibrium in a general class of auction-related games. The relation between the grade distributions and the achievement functions follows immediately from the fact that achievement is a strictly decreasing function of private cost types. A formal proof of uniqueness is a bit more involved and is under construction. Briefly though, it follows the same logic as Hickman [13, Proposition 3.3, Theorem 3.4]. Given the well-behaved nature of the type distributions, it can be shown that any symmetric, monotonic equilibrium must also be differentiable. From differentiability, it follows that the equilibrium achievement functions must satisfy the first-order conditions of an agent's objective function. The first-order conditions define a standard initial value problem, and the fundamental theorem of differential equations can be invoked to show that a unique solution exists. Since any symmetric equilibrium of the college admissions game must be consistent with the unique solution of the first-order conditions, it follows that the symmetric equilibrium is unique. ■

A.0.2. **Proof of Theorem 5.1. Proof:** For notational ease, I shall drop the "*cb*" superscripts for the duration of the proof. Also, recall that finite functions are denoted by the presence of a parameter K, whereas limiting functions lack the extra argument. I begin by ordering the sample of K prizes from lowest quality to highest, denoting the k^{th} order statistic by $p_{(k:K)}$. Since $\gamma_i(\theta;K)$ is monotonic for $i=\mathcal{M},\mathcal{N}$, the equilibrium expected payoff function in the K-player game can be written as

$$\begin{aligned}
&\Pi_i(s,\theta;K)\\
&\quad=\sum_{k=1}^{K} p_{(k:K)} \sum_{\substack{k_i\leq\min\{k,K_i\},\\ k_j=k-k_i}} \left[\binom{K_i-1}{k_i-1} F_i\left(\gamma_i^{-1}[s;K]\right)^{K_i-k_i}\left[1-F_i\left(\gamma_i^{-1}[s;K]\right)\right]^{k_i-1}\right.\\
&\qquad\qquad\qquad\left.\times\binom{K_j}{k_j} F_j\left(\gamma_j^{-1}[s;K]\right)^{K_j-k_j}\left[1-F_j\left(\gamma_j^{-1}[s;K]\right)\right]^{k_j}\right]\\
&\qquad\qquad\qquad-\mathcal{C}(s;\theta).
\end{aligned}$$

In order for a player from group i to win the k^{th} prize, it must be the case that exactly $k-1$ of his opponents have types above his own. For each opponent in his own group, this occurs with probability $1-F_i\left(\gamma_i^{-1}[s;K]\right)$, and for each opponent in the other group, this occurs with probability $1-F_j\left(\gamma_j^{-1}[s;K]\right)$. The binomial coefficients and the second summation operator in the expression above are designed to cover all the possible ways in which exactly $k-1$ opponents have higher costs. Thus, the term within the inner summation is the probability of winning the k^{th} prize, and the overall objective function is a weighted sum of all K prizes, giving the expected prize won in equilibrium.

Recall my claim that the limiting equilibrium payoff function is given by

$$\Pi(s,\theta)=F_P^{-1}\left[1-\left(\mu F_{\mathcal{M}}\left[\gamma_{\mathcal{M}}^{-1}(s)\right]+(1-\mu)F_{\mathcal{N}}\left[\gamma_{\mathcal{N}}^{-1}(s)\right]\right)\right]-\mathcal{C}(s;\theta).$$

I wish to show that for large K, it is nearly optimal to act as if one were maximizing $\Pi(s,\theta)$, rather than $\Pi(s,\theta;K)$. Since costs never change with K, I shall drop the cost terms and focus solely on convergence of the gross payoff functions $\pi(s,\theta;K)$ to their limit $\pi(s,\theta)$.

For $l \in [0,1]$, define

$$p(l;K) \equiv \left\{ p_{(t:K)} : t = \underset{k \in \{1,\dots,K\}}{\operatorname{argmin}} \left| l - \frac{k}{K} \right| \right\},^{23}$$

Intuitively, $\{p(l;K)\}_{K=1}^{\infty}$ can be thought of as a random sequence of the t^{th} order statistic in the sample of prizes, where for each K, t is chosen so that $p_{(t:K)}$ approximates the l^{th} sample quantile as closely as possible. Since $\left| l - \frac{i}{K} \right| \leq \frac{1}{2K}$ for all $l \in (0,1)$, in the limit the t^{th} order statistic will be precisely at the l^{th} quantile within the sample of K prizes. Furthermore, since the sample distribution converges to F_P by the law of large numbers, it follows that $\underset{K \to \infty}{\operatorname{plim}} p(l;K) = F_P^{-1}(l)$.

For some $i = \mathcal{M}, \mathcal{N}$, fix $\theta \in [\underline{\theta}, \overline{\theta}]$ and let

$$l = 1 - \mu F_{\mathcal{M}} \left[\gamma_{\mathcal{M}}^{-1}(s) \right] - (1-\mu) F_{\mathcal{N}} \left[\gamma_{\mathcal{N}}^{-1}(s) \right],$$

where $s = \gamma_i(\theta)$. Notice that

$$\underset{N \to \infty}{\operatorname{plim}} p(l;K) = F_P^{-1} \left(1 - \mu F_{\mathcal{M}} \left[\gamma_{\mathcal{M}}^{-1}(s) \right] - (1-\mu) F_{\mathcal{N}} \left[\gamma_{\mathcal{N}}^{-1}(s) \right] \right) = \pi(s,\theta).$$

Moreover, For each K I can rewrite the finite expected gross payoff function as

$$\begin{aligned}
\pi_i(s,\theta;K) = p(l;K) & \sum_{\substack{k_i \leq \min\{t,K_i\},\\ k_j = t - k_i}} \Bigg[\binom{K_i - 1}{k_i - 1} F_i(\theta)^{K_i - k_i} \left[1 - F_i(\theta)\right]^{k_i - 1} \\
& \times \binom{K_j}{k_j} F_j \left(\gamma_j^{-1}[s;K] \right)^{K_j - k_j} \left[1 - F_j \left(\gamma_j^{-1}[s;K] \right) \right]^{k_j} \Bigg] \\
\sum_{\substack{k=1,\dots,N,\\ k \neq t}} p_{(k:K)} & \sum_{\substack{k_i \leq \min\{k,K_i\},\\ k_j = k - k_i}} \Bigg[\binom{K_i - 1}{k_i - 1} F_i(\theta)^{K_i - k_i} \left[1 - F_i(\theta)\right]^{k_i - 1} \\
& \times \binom{K_j}{k_j} F_j \left(\gamma_j^{-1}[s;K] \right)^{K_j - k_j} \left[1 - F_j \left(\gamma_j^{-1}[s;K] \right) \right]^{k_j} \Bigg].
\end{aligned} \tag{A.1}$$

Let

$$k_i^* \equiv \underset{1 \leq k \leq \min\{t,K_i\}}{\operatorname{argmin}} \left| (1 - F_i(\theta)) - \frac{k}{K_i} \right|^{24}$$

[23] If there are multiple maximizers (there can be at most two) then choose t to be the lesser.

[24] If there are multiple maximizers (there can be at most two) then choose k_i^* to be the lesser.

and note that the following can be extracted from the first term in (A.1):

$$p(l;K)\left[\binom{K_i-1}{k_i^*-1}F_i(\theta)^{K_i-k_i^*}(1-F_i(\theta))^{k_i^*-1}\right]$$
$$\times\left[\binom{K_j}{t-k_i^*}F_j\left(\gamma_j^{-1}[s;K]\right)^{K_j-t-k_i^*}\left[1-F_j\left(\gamma_j^{-1}[s;K]\right)\right]^{t-k_i^*}\right].$$

The second and third components of the above product represent the probability that exactly $K_i-k_i^*$ group-i players have costs below θ and exactly $K_j-t-k_i^*$ group-j players achieve grades below $\gamma_i(\theta;K)$. Letting

$$\mu_i=\begin{cases}(1-\mu) & i=\mathcal{N}\quad\text{and}\\ \mu & i=\mathcal{M},\end{cases}$$

this can be restated as the probability that fraction

$$\frac{K_i-k_i^*}{K_i}=1-\frac{k_i^*}{K_i}\xrightarrow{K}F_i(\theta)$$

of group-i players have costs below θ and fraction

$$\begin{aligned}\frac{K_j-t+k_i^*}{K}&\xrightarrow{K}\mu_j-l+\mu_i(1-F_i(\theta))\\ &=\mu_j-1+(1-\mu)_iF_i(\theta)+\mu_jF_j(\gamma_j^{-1}[s;K])+\mu_i(1-F_i(\theta))\\ &=\mu_jF_j(\gamma_j^{-1}[s;K])\end{aligned}$$

of all agents come from group j and achieve equilibrium grades below $\gamma_i(\theta;K)$. In each of the previous two expressions, the convergence over K term follows from the law of large numbers. Since the probability associated with this event is one in the limit, it follows that the pointwise probability limit of (A.1) is $\pi(s,\theta)$, for $i=\mathcal{M},\mathcal{N}$.

Given that $\{\Pi(s,\theta;K)\}_{K=1}^{\infty}$ is a sequence of measurable functions converging pointwise to $\Pi(s,\theta)$ on a measurable set of finite measure, by Egorov's Theorem it follows that for any $\rho>0$ there exists a set $E\subset[\underline{\theta},\overline{\theta}]$ having measure $m(E)<\rho$, such that $\{\Pi(s,\theta;K)\}_{K=1}^{\infty}\rightarrow\Pi(s,\theta)$ uniformly on the set $[\underline{\theta},\overline{\theta}]\setminus E$.

This is the same as saying that on the set $[\underline{\theta},\overline{\theta}]\setminus E$, it is nearly optimal to choose one's bid as if one's opponents were adopting a strategy of $\gamma(\theta)$, rather than $\gamma(\theta;K)$. Thus, given $\varepsilon>0$, there exists K_ε such that for any $K\geq K_\varepsilon$, $\gamma(\theta)$ generates an ε-equilibrium of the K-player finite game. Furthermore, since all of the model primitives are well-behaved–θ is strictly bounded away from zero; $\mathcal{P}$ is compact; $F_{\mathcal{M}}$, $F_{\mathcal{N}}$, and F_P are absolutely continuous; and for each θ the set of undominated bids is compact-valued–I can invoke the Theorem of the Maximum on any compact subset of $[\underline{\theta},\overline{\theta}]\setminus E$ to show that the maximizers of $\Pi(s,\theta;K)$ and $\Pi(s,\theta)$ are close for large K. That is, given $\delta>0$, there exists K_δ such that for any $K \geq K_\delta$, $\gamma(\theta)$ is a δ-approximate equilibrium of the K-player finite game, or

$$\|\gamma(\theta) - \gamma(\theta; K)\|_{\sup} < \delta.$$

Finally, given $\varepsilon > 0$ and $\delta > 0$, then for any $K \geq K^* \equiv \max\{K_\varepsilon, K_\delta\}$, $\gamma(\theta)$ is a δ-approximate equilibrium which generates an ε-equilibrium of the K-player finite game on any closed subset of $[\underline{\theta}, \overline{\theta}] \setminus E$. ■

A.0.3. **Proof of Theorem 5.2. Proof:** The logic of the proof is very similar to that of Theorem 5.1, but it is simpler because there is only one distribution to work with. Once again, I drop the "q" superscripts for the remainder of the proof and I begin by ordering the random sample of K prizes from lowest quality to highest, denoting the k^{th} order statistic by $p_{(k:K)}$. Since $\gamma(\theta; K)$ is monotonic, the equilibrium expected payoff function in the K-player game can be written as

$$\Pi(s, \theta; K) = \sum_{k=1}^{K} p_{(k:K)} \left[\binom{K-1}{k-1} F_i(\theta)^{K-k} (1 - F_i(\theta))^{k-1} \right] - \mathcal{C}(s; \theta).$$

The first term is a weighted average of the order statistics, where the weights are the probabilities of winning each prize.[25] Recall my claim that the limiting equilibrium payoff function is given by

$$\Pi(s, \theta) = F_P^{-1} \left(1 - F_i(\gamma_i^{-1}(s)) \right) - \mathcal{C}(s; \theta).$$

I wish to show that for large K, it is nearly optimal to bid as if one were maximizing $\Pi(s, \theta)$, rather than $\Pi(s, \theta; K)$. Since the cost of submitting a given bid never changes, I drop the second term from each payoff function and focus on convergence of the reward function sequence $\{\pi(\theta; K)\}_{K=1}^{\infty}$ to its limit $\pi(\theta)$.

For $l \in [0, 1]$, define

$$p(l; K) \equiv \left\{ p_{(t:K)} : t = \underset{k \in \{1, \ldots, K\}}{\operatorname{argmin}} \left| l - \frac{i}{K} \right| \right\},$$ [26]

and once again, $\{p(l; K)\}_{K=1}^{\infty}$ can be thought of as a random sequence of the t^{th} order statistic in the sample of prizes, where for each K, t is chosen so that $p_{(t:K)}$ approximates the l^{th} sample quantile as closely as possible. Note that by the same logic as in the proof

[25] In order for a player to win the k^{th} prize, there must be exactly $K - k$ competitors with lower costs and $k - 1$ with higher costs. The probabilities of these two events are $F_i(\theta)^{K-k}$ and $(1 - F_i(\theta))^{k-1}$, respectively. Finally, there are $\binom{K-1}{k-1}$ ways in which the intersection of the two events can occur. Thus, the probability of winning the k^{th} prize is $\binom{K-1}{k-1} F_i(\theta)^{K-k} (1 - F_i(\theta))^{k-1}$.

[26] If there are multiple maximizers (there can be at most two) then choose t to be the lesser.

of Theorem 5.1, we have $\underset{N\to\infty}{\text{plim}}\, p(l;K) = F_P^{-1}(l)$. Fix $\theta \in [\underline{\theta}, \overline{\theta}]$ and let $l = 1 - F_i(\theta)$. Notice that

$$\underset{K\to\infty}{\text{plim}}\, p(l;K) = F_P^{-1}(1 - F_i(\theta)) = \pi(s,\theta).$$

Moreover, For each K I can rewrite the finite expected gross payoff function as

$$\begin{aligned}
\pi(s,\theta;K) =& p(l;K)\left[\binom{K-1}{t-1}F_i(\theta)^{K-t}(1-F_i(\theta))^{t-1}\right] \\
&+ \sum_{k=1}^{t-1} p_{(k:K)}\left[\binom{K-1}{k-1}F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1}\right] \\
&+ \sum_{k=t+1}^{K} p_{(k:K)}\left[\binom{K-1}{k-1}F_i(\theta)^{K-k}(1-F_i(\theta))^{k-1}\right].
\end{aligned} \tag{A.2}$$

Note that

$$\begin{aligned}
\left[\binom{K-1}{t-1}F_i(\theta)^{K-t}(1-F_i(\theta))^{t-1}\right] &= \Pr[K-t \text{ competitors have costs less than } \theta] \\
&= \Pr[\text{fraction } \frac{K-t}{K} \text{ have lower costs}] \\
&= \Pr[\text{fraction } \left(1-\frac{t}{K}\right) \text{ have lower costs}] \\
&\xrightarrow{K} \Pr[\text{fraction } (1-l) \text{ have lower costs}] \\
&= \Pr[\text{fraction } F_i(\theta) \text{ have lower costs}] \\
&=1,
\end{aligned}$$

where the convergence over K follows from the law of large numbers. Since probabilities must sum to one, equation (A.2) reveals that $\pi(\theta;K)$ increasingly resembles $p(l;K)$ as K gets large. Furthermore, since $\underset{N\to\infty}{\text{plim}}\, p(l;K) = F_P^{-1}(l)$, it follows that the pointwise probability limit of $\pi(\theta;K)$ is $\pi(\theta)$.

With pointwise convergence out of the way, the remainder of the proof is identical to the second half of the proof of Theorem 5.1. ■

A.0.4. **Alternative Proof of Equilibrium Approximation.** As mentioned in the body of the paper, the various results on equilibrium approximation can be strengthened to demonstrate that the derivations accurately reflect equilibrium actions and outcomes on the entire support of types. The cost of the stronger result is application of a more complicated proof which invokes results that may be less familiar to economic theorists. The logic is very similar under all three cases of color-blind admissions, quotas and admission preferences, so here I merely state and prove the alternative claim in the case of a quota rule, where the notation is simplest.

Theorem A.0.1. *Given $\varepsilon,\ \delta > 0$, there exists $K^* \in \mathbb{N}$, such that in the college admission game with a quota rule, we have the following:*

(i) $\gamma_i^q(\theta),\ i = \mathcal{M}, \mathcal{N}$ *as defined by equation (5) and boundary condition (3) generates an ε-equilibrium of the K-player quota game, and*

(ii) $\gamma_i^q(\theta)$ *is a δ-approximate equilibrium for the K-player quota game, or*

$$\|\gamma_i^q(\theta) - \gamma_i^q(\theta; K)\|_{\sup} < \delta,\ \ i = \mathcal{M}, \mathcal{N}.$$

Proof: The first part of the proof involves showing that the finite objective functions converge pointwise in probability to the proposed limiting objective. The argument is identical to the one in the first half of the proof of Theorem 5.2

Using pointwise convergence, I can then invoke Newey [17, Theorem 2.1] uniform convergence theorem to show that $\tilde{\Pi}(\theta; K)$ converges uniformly in probability to $\Pi(\theta)$ on the entire interval $[\underline{\theta}, \overline{\theta}]$. In order to do so, I must first verify a regularity condition and *stochastic equicontinuity* of the sequence $\tilde{\Pi}(\theta; K)$. For this part of the argument, it will be easier to think in terms of l, rather than θ. The regularity condition is that $l(\theta) = 1 - F_i(\theta)$ must live on a compact interval. Since θ lives on a compact interval and since F_i is continuous and monotonic, $l(\theta)$ attains values of 0 and 1 for finite values of θ.

At this point, all that remains is to verify equicontinuity of the sequence of functions $\tilde{\Pi}(\theta; K)$. For deterministic functions, it is known that pointwise convergence on a compact interval to a continuous limit implies uniform convergence if the sequence is equicontinuous. There is also an analogous condition for sequences of random functions, known as stochastic equicontinuity. In the context of my model, it basically means that for any point θ, $\tilde{\Pi}(\theta; K)$ must be continuous at θ at least with probability close to one for large K.[27] More precisely, $\{\tilde{\Pi}(\theta; K)\}_{K=1}^{\infty}$ is stochastically equicontinuous if for any $\epsilon, \epsilon' > 0$ there exists $\tau > 0$ such that

$$\begin{aligned}
&\limsup_{K\to\infty} \Pr\left[\sup_{l\in[0,1],\ l'\in B_\tau(l)} \left|\tilde{\Pi}(l; K) - \tilde{\Pi}(l'; K)\right| > \epsilon'\right] \\
&= \limsup_{K\to\infty} \Pr\Bigg[\sup_{l\in[0,1],\ l'\in B_\tau(l)} \Bigg| \sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l)^{K-i}(l)^{i-1}\right] \\
&\qquad\qquad - \sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1}(1-l')^{K-i}(l')^{i-1}\right] \Bigg| > \epsilon'\Bigg] < \epsilon,
\end{aligned}$$

where $B_\tau(l)$ is an open ball centered at l with radius τ.

[27] For a more detailed discussion on stochastic equicontinuity, see Andrews [1, Section 2.1].

By similar arguments as above, it is apparent that

$$\sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1} (1-l)^{K-i} (l)^{i-1} \right] \to F_P^{-1}(l) \text{ and}$$

$$\sum_{i=1}^{K} p_{(k:K)} \left[\binom{K-1}{i-1} (1-l')^{K-i} (l')^{i-1} \right] \to F_P^{-1}(l').$$

Therefore, I can satisfy stochastic equicontinuity by choosing τ^* so that for all $l \in [0,1]$ and $l' \in B_{\tau^*}(l)$, the following is true:

$$\left| F_P^{-1}(l) - F_P^{-1}(l') \right| < \epsilon'.$$

Since F_P is continuous and $\mathcal{P}$ is compact, such a τ^* indeed exists. Thus, by Newey's uniform convergence theorem, it follows that for all $\epsilon > 0$, I have

$$\lim_{K \to \infty} \Pr \left[\|\tilde{\Pi}(\theta; K) - \Pi(\theta)\|_{\sup} > \epsilon \right] = 0.$$

In other words, When K is large, the equilibrium grade distribution under a monotonic equilibrium is such that it is nearly optimal to maximize as if one's (equilibrium) objective function were $\Pi(\theta, s) = F_P^{-1}(1 - F_i(\theta)) - \mathcal{C}(s; \theta)$.

This is the same as saying that it is nearly optimal to choose one's grade as if one's opponents were adopting a strategy of $\gamma(\theta)$, rather than $\gamma(\theta; K)$. Thus, given $\varepsilon > 0$, there exists K_ε such that for any $K \geq K_\varepsilon$, $\gamma(\theta)$ generates an ε-equilibrium of the K-player finite game. Furthermore, since all of the model primitives are well-behaved–θ is strictly bounded away from zero and lives in a compact set, $\mathcal{P}$ is compact, F_i and F_P are absolutely continuous and for each θ the set of undominated grades is compact-valued–the Theorem of the Maximum implies that the maximizers of $\tilde{\Pi}(s, \theta; K)$ and $\Pi(s, \theta)$ are close for large K. That is, given $\delta > 0$, there exists K_δ such that for any $K \geq K_\delta$, $\gamma(\theta)$ is a δ-approximate equilibrium of the K-player finite game, or

$$\|\gamma(\theta) - \gamma(\theta; K)\|_{\sup} < \delta.$$

Finally, given $\varepsilon > 0$ and $\delta > 0$, then for any $K \geq K^* \equiv \max\{K_\varepsilon, K_\delta\}$, $\gamma(\theta)$ is a δ-approximate equilibrium which generates an ε-equilibrium of the K-player finite game. ■

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