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# Decision Support for Consumer Direct Grocery Initiatives

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Many companies with consumer direct service models, especially grocery delivery services, have found that home delivery poses an enormous logistical challenge due to the unpredictability of demand coupled with strict delivery windows and low profit margin products. These systems have proven difficult to manage effectively and could benefit from new technology, particularly to manage the interaction between order capture and order delivery. In this article, we define routing and scheduling problems that incorporate important features of this emerging business model and propose algorithms, based on insertion heuristics, for their solution. In the proposed home delivery problem, the company decides which deliveries to accept or reject as well as the time slot for the accepted deliveries so as to maximize expected profits. Computational experiments reveal the importance of an approach that integrates order capture with order delivery and demonstrates the quality and value of the proposed algorithms.

*Key words*: e-commerce; home delivery; real-time decision making; vehicle routing; insertion heuristics; delivery windows
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## 1. Introduction

Business-to-consumer e-commerce has not only increased the demand for package delivery services, such as those offered by Federal Express and United Parcel Service (UPS), but has also led to the development of new consumer direct (CD) service models and activities such as grocery delivery services. By bringing goods "the last mile" to the customer's front door, home delivery is seen by many as the ultimate value-added service for consumers. It is expected that in the future we will see a proliferation of new delivery services and options. Forrester Research expects online purchases by U.S. consumers to grow to $184 billion by 2004, or seven percent of total retail spending that year (Jardine Logistics Report 2001). Many of the companies that have started to provide home delivery services have found, however, that direct delivery poses an enormous logistical challenge and are struggling to create direct delivery strategies that are profitable and can satisfy customer expectations. For example, the difficult challenge of matching grocery store prices while incurring high distribution costs led to closing of the San Francisco-based operations for Peapod (Cox 2001) and the declaration of bankruptcy by Webvan (Farmer and Sandoval 2001).

CD is obviously not limited to online grocery services. In many other industries, CD is considered an improved service model. For example, it is becoming more and more common for drug companies to offer home delivery services, e.g., CVS ProCare (www.stadtlander.com) and CIGNA Tel-Drug (www.teldrug.healthcare.cigna.com). Industries with established service practices have also adapted to using the Internet. For example, in the telecommunication industry, AT&T allows customers to set up appointments for cable and phone service installations through www.att.com. Other examples of companies providing CD delivery services include Staples (www.staples.com), which delivers office supplies to business consumers. In fact, Staples in Canada partnered with a grocery delivery service, since office supplies and groceries have different peak demand times.

The fulfillment process for most consumer direct businesses can be divided into three phases: (1) order capture and promise, (2) order sourcing and assembly, and (3) order delivery. Efficient order sourcing and assembly can be rather difficult, with most orders involving roughly 50 individual stock keeping units (SKUs) with multiple temperature requirements, but this research effort focuses primarily on the interactions between order promise (deciding on a delivery time) and order delivery (devising efficient delivery schedules). Better integration of order promise and order delivery decisions has the potential to substantially improve profitability, especially for those

CD businesses offering "attended" deliveries. Attended deliveries are those where the consumers must be present and may be necessary for security reasons (e.g., expensive computer equipment), because goods are perishable (e.g., milk or flowers) or because goods are being picked up or exchanged with the consumer (e.g., dry cleaning, videos/DVDs) and are a vital feature of many CD service models. To avoid delivery failures as much as possible, it is customary in attended home delivery services (HDS) for the company and customer to mutually agree on a narrow delivery window or time slot.

These stringent time restrictions, combined with unpredictable demand and the pressure to control costs, create quite a challenge. Companies are quickly learning that unless they can set up an effective and efficient HDS, the cost savings promised by e-commerce will continue to be eaten up by high delivery costs, leaving them struggling for profitability. With hindsight, for example, it is easy to observe that the prices consumers initially paid for home grocery delivery services were not in line with the costs associated with order processing and distribution. Companies offered free delivery, equivalent prices to those found in supermarkets, and sometimes even free gifts, in hopes that the resulting high quantity of deliveries would be enough to make them profitable. Given the less-than-expected demand, the complex and multifaceted distribution problem created by home delivery (especially with the 30-minute time windows promised initially by Webvan), and the low-margin consumer goods associated with grocery delivery, this has not proved to be a successful strategy.

In this article, we propose and analyze various routing and scheduling problems and algorithms that will lead to more effective decision technology to support HDS, especially for CD full-line grocery services. Home delivery is a fairly new phenomenon, and thus few models and algorithms have been proposed and studied that help create an understanding of the complexities and intricacies of these distribution problems. Our goal is to change that situation. One of the contributions of this article is the introduction of the home delivery problem, a new problem that captures many of the core features of HDS and allows the research community to focus on a common problem. In this problem, the vendor decides which deliveries to accept or reject and the time slot for the delivery, if it is accepted, to maximize expected profits. We propose an algorithm, based on insertion techniques, to help make these decisions. Another contribution of this work is that it provides a demonstration of the importance of considering opportunity costs associated with accepting a delivery request in a certain time slot. The final contribution of the article is that it sets the stage for the integration of profit optimization into the developed models and techniques by defining the home delivery problem with incentives.

The article is organized as follows. Section 2 provides a short discussion of current practice in home grocery delivery services. In §3, the relevant literature is summarized, and in §4, the home delivery problem is introduced and defined. We develop insertion-based heuristics for the solution of instances of the home delivery problem in §5. In §6, the results of a set of computational experiments are presented. Finally, in §7, we comment on the presented research and discuss future plans.

## 2. Current Practice

It is current practice in home grocery delivery models for the retailer and consumer to agree on a delivery time slot and for these time slots to be available on a first-come, first-served basis. Often the retailer accepts a fixed number of requests per time slot, where this number is typically based on a combination of factors, including fleet size and historic delivery times. If the maximum number of deliveries accepted per time slot is three, for example, then, after a particular time slot is selected for the third time, it is no longer presented to the consumer as a viable delivery time slot. In the computational experiments, this approach will be referred to as SLOT.

Some observations are in order. First, the closer the delivery locations are for orders in a given time slot (or consecutive time slots), the easier it will be to schedule the deliveries and the cheaper it will be to carry them out. Second, the closer the deliveries are in a given time slot, the more deliveries can be accepted and effectively handled, which reduces average delivery costs and increases profitability.

These observations set the stage for improving the profitability of home delivery operations. The next generation of decision support technology should

- dynamically determine whether a delivery request can still be accommodated based on a set of tentative routes for already accepted customers and the location of the customer in question, and
- consider the opportunity cost associated with accepting the customer in question in a given time slot, in view of the fact that more profitable delivery requests may still arrive.

After reviewing the relevant literature, we formally propose the home delivery problem (HDP), which will allow us to study these desirable algorithmic properties.

## 3. Literature

Research is emerging that analyzes routing strategies for unattended home deliveries where time slots are not of concern. In Punakivi (2000), the routing options studied include the use of fixed routes and the optimal sequencing of the deliveries on routes as soon as all deliveries are known. The results demonstrate the importance of optimization in CD with savings from optimal routing versus fixed routing of 54% for sparse areas. For dense areas, the cost savings averaged 18%. Yrjölä (2001) concludes that attended deliveries can often cost more than double that of unattended deliveries. Saranen and Småros (2001) simulated the delivery costs for two specific models—Streamline.com's unattended delivery policy and Webvan's attended half-hour delivery window policy—and found the more restrictive Webvan model to cost five times more.

The key decision in the HDP to be defined in the next section, where all deliveries are attended, is whether to accept or reject a delivery request as it arrives and, if accepted, the choice of the delivery time slot. There are obvious similarities with airline revenue management. Airline revenue management focuses on how to evaluate whether accepting a reservation for a seat in a certain fare class displaces capacity that could be used for reservations in higher fare classes. However, there are also important differences with airline revenue management. The costs associated with an airline seat do not depend on who occupies the seat or on who purchases the remaining tickets in the same fare class. On the other hand, the costs associated with accepting an order in a certain time slot depend on the delivery address, the time slot, and the delivery locations and time slots of the other accepted orders. Therefore, computing the expected profitability of accepting a delivery request in a certain time slot has to involve the solution of vehicle routing problems with time windows. Given that decisions have to be made in real time, the routes must be created quickly and must properly discount revenue and costs for anticipated orders. Some of these concepts are addressed in the stochastic and dynamic routing literature.

In stochastic routing, there are two research streams. The first stream focuses on the vehicle routing problem with stochastic demands (VRPSD) (Bertsimas, Jaillet, and Odoni 1990; Bertsimas 1992; Bertsimas and Simchi-Levi 1996; Gendreau, Laporte, and Seguin 1995, 1996). The second stream focuses on the vehicle routing problem with stochastic customers (VRPSC) (Gendreau, Laporte, and Seguin 1996). The VRPSD focuses on situations in which the size of customer demand is unknown (usually modeled as normally distributed with known mean and variance). The VRPSC focuses on situations where the size of customers demand is not the issue, but whether the demand occurs (usually modeled as a fixed probability). Both the VRPSD and the VRPSC are solved by creating a set of routes that minimizes expected costs before the demands are realized. As demands are realized, they are automatically assigned to a route and a position on the route according to the *a priori* schedule. The expected costs include the travel costs between the customers on the routes as well as the penalty costs created if additional routes are needed when more demand is realized than can be served by the assigned vehicle.

In dynamic routing, there are also two research streams. The first stream focuses on building tentative schedules to minimize costs based solely on the orders that have already been accepted, e.g., the techniques described in Bagchi and Nag (1991), Prosser, Kilby, and Shaw (1998), Psaraftis (1988, 1995), Rego and Roucairol (1995), Savelsbergh and Sol (1998), and Zhu and Ong (2000). The second stream focuses on minimizing the costs, considering not only the orders that have already been accepted but also the expected future demand, as in Cheung and Powell (1996), Frantzekakis and Powell (1990), Powell (1996), and Regan, Mahmassani, and Jaillet (1998a, b). The latter papers are concerned with trucking companies and focus on whether to accept or reject a demand request, which consists of both a pickup and delivery point, and on how to assign each accepted task to a vehicle to maximize profit for the long term. Since the primary decision is where to send the vehicles next, stochastic information is used to evaluate the probability that new orders are available at or near the destination of the task being executed by the time of its arrival at the destination. Usually the models assume that the origin and destination of a task are cities (or other aggregated areas) large enough to approximate the number of forecasted demand request by an integer greater than one.

As indicated above, revenue management is widely used in the airline industry to control overbooking and allocation of seats among different price classes. The choice of how many seats on each flight should be sold at which fares is based on minimizing lost opportunities to sell full-fare seats and maximizing revenue. Decision models use the estimated probability of receiving the full fare for the seat to compare expected full-fare revenue with the return from discounted fares (Littlewood 1972). These probability calculations incorporate future expected demand, information about the accuracy of demand forecasts, and estimates of the so-called sell-up probability. More recent trends involve the dynamic computation of bid prices to determine cutoffs in making decisions whether to accept or reject current bids on ticket prices (Talluri and Ryzin 1998, 1999). The obvious and crucial difference with home delivery is the absence of a varying cost structure. In airline revenue management, all plane itineraries are known and fixed, so the major issue is how to price the seats, given predictions of future demand, in order to maximize revenue. In the home delivery context, accepting an order not only changes the delivery route and thus the delivery costs, but it may also change the costs associated with any anticipated orders.

## 4. The HDP

We will now define the dynamic routing and scheduling problem studied in this article, which we refer to as the HDP.

A set of delivery routes is constructed for a specific day in the not-too-distant future. Requests from a known set of customers for a delivery on that particular day arrive in real time and are considered up to a certain cutoff time, which precedes the actual execution of the planned delivery routes. Each delivery request has to be accepted or rejected when it arrives. If accepted, a request from customer $i$ consumes $d_i$ of vehicle capacity and results in a revenue of $r_i$. There is a homogeneous set of $m$ vehicles with capacity $Q$ to serve the accepted orders. At each point in time $t$, customer $i$ will place a request for delivery between $t$ and the cutoff time with probability $p_i(t)$. The requests are equally likely to occur at any point in time, so there is no information on the order in which the requests may be realized. The objective is to maximize the total profit resulting from executing the set of delivery routes, i.e., total revenues minus total costs (assumed to depend linearly on the total distance traveled). To increase the level of service, each delivery is guaranteed to take place during a one-hour time slot on the day of delivery. The one-hour delivery time slots are nonoverlapping and cover the entire day, e.g., 8:00–9:00, 9:00–10:00, ..., 19:00–20:00. For each customer, a time slot profile identifies which time slots are acceptable for delivery. At the time a request is accepted, a commitment to delivery during a specific acceptable time slot is also made.

Note that this problem can also be viewed as follows. When a delivery request arrives, the HDS provider determines all viable time slots for delivery and displays only those to the customer. The customer then chooses one or decides to withdraw the delivery request. This view is more closely related to the HDS found in practice.

A "customer" in this context can represent one address, such as a regular customer, or the aggregated demand from a compact space such as a city block. As indicated in the problem definition, all orders are received prior to the execution of any delivery. That is, we will not consider "same-day delivery" services where orders arrive during the execution of a delivery schedule and have to be incorporated immediately. We feel this is justified, because in many consumer home delivery environments the vehicles will be loaded with customer-specific orders, so once a vehicle has started its route it cannot be rerouted to a new customer since the correct inventory will likely not be on board.

## 5. Solving the HDP

As a first step toward developing a solution approach for the HDP, we start by developing technology to dynamically determine whether a delivery request—characterized by a size, a delivery address, and a set of acceptable time slots—can still be accommodated based on the set of already accepted customers. Doing this should increase the number of delivery requests that can be accepted and feasibly delivered. To dynamically determine whether a delivery request can be accommodated, we have to determine if there exists a set of routes visiting all previously accepted deliveries as well as the delivery request under consideration, and we have to do so quickly. If such a set of delivery routes exists, then the new request can be accepted; if no feasible set of delivery routes exists, then the new request has to be rejected.

It is well known that deciding whether a feasible solution to the vehicle routing problem with time windows exists is NP-complete (Savelsbergh 1986), so it is natural to consider employing heuristics to answer the question quickly. We have chosen to use an insertion heuristic to solve the resulting vehicle routing problem with time windows. Our insertion heuristic consists of two phases. In the first phase, all accepted delivery requests are inserted using the following insertion criterion (assuming customer $j$ is being inserted between two consecutive customers $i-1$ and $i$ in an existing route)

$$r_j - (c_{i-1,\,j} + c_{j,\,i} - c_{i-1,\,i}),$$

where, because we are trying to maximize profit, the customer with the largest criterion value is always selected as the one to be inserted next. In the second phase, we evaluate whether the delivery request under consideration can be inserted in one of these partially constructed routes during one of the time slots acceptable for the associated customer. This order of insertions is important to ensure that the delivery request under consideration does not prevent any of the previously accepted deliveries from being completed during its committed time slot.

To further improve the chances that the delivery request under consideration can be inserted, randomization is used during construction in the form of a greedy randomized adaptive search procedure (GRASP) (Kontoravdis and Bard 1995). We choose randomly from the top $k$ most profitable insertions at each decision point in Phase 1. This enables us to create several different sets of delivery routes for the already accepted deliveries and use each of these to see if there is a feasible insertion for the delivery request under consideration. In case there are multiple sets of delivery routes in which the delivery request under consideration can be inserted, we select the set of delivery routes with the highest profit and commit to the time slot that the delivery request under consideration occupies. In the computational experiments, this approach will be referred to as *DYN*.

The approach discussed above for determining whether an incoming delivery request can be accommodated completely ignores the fact that additional, more profitable requests may still arrive, i.e., requests that have a higher revenue or that can be incorporated more cost effectively because of their location.

When a delivery request materializes, one has to decide whether to accept it or not and, if it is accepted, which of the time slots in the customer's time slot profile to assign to it. To consider the future in the decision technology, we need to compute the expected total profit if the request is rejected, as well as the expected total profit if the request is accepted in each of the time slots in the customer's profile.

Conceptually, this can be done by computing for every possible realization of future delivery requests the following values:

- the probability that this realization of future delivery requests occurs,
- the optimal solution value for the resulting vehicle routing with time windows (VRPTW) solution *without* the request under consideration (but with all already accepted requests and all future realized requests), and
- for each time slot in the time slot profile associated with the request under consideration, the value of the optimal VRPTW solution *with* the request served during the specified time slot (along with all of the already accepted requests and all future realized requests).

With this information we can compute the expected total profit if the request is rejected or accepted in each of the time slots in the customer's profile. If the expected total profit without the request is larger than the expected total profit with the request for all time slots, then the request will be rejected. Otherwise, the request will be accepted and the time slot that resulted in the largest expected total profit will be assigned to it.

Unfortunately, this approach is computationally intractable because the number of possible future realizations is too large and because the VRPTW is an NP-hard problem. To develop a computationally tractable approach, we try to approximate the above conceptual solution approach through an insertion heuristic that solves a single instance of a modified VRPTW each time a request materializes. The created instance of the VRPTW includes all already accepted requests, the request currently under consideration, and all requests that may or may not materialize in the future. The objective is to maximize profit where we accept that it may not be possible to satisfy all requests due to limited capacity or time. (Note that this is different from traditional vehicle routing problems, in which it is implicitly assumed that all requests can be delivered.) If the request under consideration is part of the constructed set of delivery routes, it is more valuable to include this request rather than to wait for future requests, so it is accepted; if the request under consideration is not part of the constructed set of delivery routes, it is rejected. To account for the differences in customer status—i.e., some requests have already been accepted and others have not yet materialized—we adjust the revenue and the capacity requirements of the requests that have not materialized yet based on the probability that a delivery request will be received before the cutoff time.

The insertion heuristic consists of two phases. In the first phase, all accepted delivery requests are inserted using the criterion (assuming customer $j$ is being inserted between two consecutive customers $i-1$ and $i$ in an existing route)

$$r_j - (c_{i-1,j} + c_{j,i} - c_{i-1,i}),$$

where, because we are trying to maximize profit, the customer with the largest criterion value is always selected as the one to be inserted next. In the second phase, the remaining customers are inserted until there are no more feasible insertions because of limited capacity. As mentioned above, the size of each delivery request in the second phase is adjusted downward by its probability of being realized; i.e., the size is set to $p_i(t)d_i$ for request $i$ if we are currently at time $t$. Note that the request currently under consideration is inserted in the second phase.

We have experimented with four different insertion criteria to evaluate the potential profitability associated with an insertion in the second phase.

- *DSR*: To account for the fact that we may be dealing with a yet-unrealized delivery request, we use the expected revenue associated with a request, i.e.,

$$p_j(t)r_j - (c_{i-1,j} + c_{j,i} - c_{i-1,i}).$$

This criterion simply biases the insertion heuristic toward delivery requests with a higher probability of being realized.

- *PATH*: In computing the cost of an insertion, the above criterion ignores the fact that the neighboring customers $i-1$ and $i$ may or may not be present in the final set of delivery routes. It may be better to compute the cost of an insertion relative to two already accepted requests, say $u$ and $v$. The expected length of the path between $u$ and $v$, assuming route $(1, \ldots, u, \ldots, v, \ldots, n+1)$, can be computed as follows:

$$\sum_{i=u}^{v-1} \sum_{k=u+1}^{v} d_i p_i(t) p_k(t) \prod_{l=i+1}^{k-1} (1 - p_l(t)).$$

We now calculate the expected length with and without request $j$ as part of the path between $u$ and $v$ and take the difference between these two values as the cost for inserting $j$. The expected revenue $p_j(t)r_j$ minus this cost yields the value of the insertion. This calculation is similar to what has been used to estimate the cost of a section of an a priori route in the stochastic vehicle routing problem (Bertsimas, Jaillet, and Odoni 1990; Bertsimas 1992). The only real difference is the penalty portion in the cost of the a priori route, representing the cost of extra trips if the sum of realized demands on the a priori route exceeds vehicle capacity. Because we accept only delivery requests that can be served feasibly, no such penalty term is required.

• *DIFF*: This criterion can be viewed as an estimate and/or simplification of the preceding criterion, which can be computed more efficiently. Both an optimistic and a pessimistic cost of the insertion are considered, represented by values $v_1$ and $v_2$, respectively. The first value is based on the current set of delivery routes

$$v_1 = c_{i-1,j} + c_{j,i} - c_{i-1,i},$$

where we evaluate inserting customer $j$ between two consecutive customers $i-1$ and $i$ in a route. The second value is based on the set of delivery routes for accepted customers only, i.e., the set of routes at the end of Phase 1

$$v_2 = c_{k-1,j} + c_{j,k} - c_{k-1,k},$$

where we evaluate inserting customer $j$ between two consecutive customers $k-1$ and $k$ in a route. The value $v_1$ represents an optimistic view on the insertion cost, because it assumes that at least some of the as-yet-unrealized delivery requests on the route will materialize, and $v_2$ represents a pessimistic view on the insertion cost, because it assumes that none of the as-yet-unrealized delivery requests on the route will materialize. The value of the insertion for request $j$ is set to

$$p_j(t)r_j - (v_2 - p_{i-1}(t)p_i(t)(v_2 - v_1)).$$

• *REG*: The first three insertion criteria ignore any synergetic effects that may occur due to close proximity of several requests that have not yet materialized. The following insertion criterion, which extends the first criterion, tries to capture that aspect,

$$\sum_{k \in R(j)} p_k(t) r_k - (c_{i-1,j} + c_{j,i} - c_{i-1,i}),$$

where $R(j)$ is some region around request $j$. Note that only unrealized requests in the region are considered.

In the discussion above, we have implicitly assumed that it is always profitable to serve a delivery request, e.g., $p_j(t)r_j - (c_{i-1,j} + c_{j,i} - c_{i-1,i}) \geq 0$. Of course, this will not always be true. To be able to handle such situations, we have adopted the following approach. *Always* insert the most profitable (and feasible) delivery request even if at that time the associated profit is negative. Note that once other (possibly nearby) unrealized delivery requests have been incorporated in the route, the profitability of a delivery request often will increase because the added delivery cost is reduced (as it is shared among the other deliveries on the route). However, to ensure that the final set of delivery routes does not include unprofitable requests, we examine the request under consideration after all unrealized deliveries have been inserted to see if its removal increases the expected total profit; if so, it is removed and the delivery is rejected.

## 6. Computational Experiments

Our primary goal in this section is to conduct computational experiments to determine the impact on total profit of using more sophisticated techniques to decide whether to accept or reject a delivery request. Furthermore, we want to study and compare the proposed insertion criteria to see if one of them outperforms and dominates the others. Finally, we want to analyze the impact of instance characteristics on the performance of our proposed technology.

### 6.1. Experimental Design

Such analysis can only be performed by means of simulation. Simulation is used to generate a stream of delivery requests at different points in time between 0 and the cutoff time $T$. Given this stream of arrivals of delivery requests, we can evaluate the behavior of the different methods by using them to decide whether to accept a request. Recall that if a delivery is rejected, it is lost forever. Also note that once the stream of delivery requests is generated, we can compute the total profit that would have resulted if the set of realized requests had been known in advance, and we can use this value to gauge the quality of our solutions. Generation of the stream of requests is guided by the probability $p_i(t)$ that customer $i$ will place a request for delivery between $t$ and $T$. For our computational experiments, a simple linearly declining probability is used, given by $p_i(T-t)/T$, where $0 < p_i \leq 1$ is specified as part of the instance.

In all our experiments, we evaluate the results over both a sparse and dense grid. For the dense grid, 100 customers are distributed over an area of 30 minutes by 30 minutes in travel time. For the sparse grid, 100 customers are distributed over an area of 60 minutes by 60 minutes in travel time. It is assumed that a vehicle travels one unit of distance per minute, and distance between customers is based on Euclidean distance calculations. We expect that the distribution costs are impacted significantly by the density of the customer locations, and these different grids allow us to analyze this impact empirically.

### 6.2. Computational Results

In this section, we present several tables of results. In each results table, statistics are provided for each grid, based on 10 randomly generated instances for the $30 \times 30$ grid and 10 instances for the $60 \times 60$ grid and for different demand probabilities, as indicated. We present the following information and statistics.

- DEC type: This specifies what method was used to decide whether to accept or reject an incoming delivery request, i.e., DYN, DSR, PATH, DIFF, or REG. For comparison purposes, we also include the results obtained when simulating current practice, denoted by SLOT, and when running the insertion heuristic with "perfect information," i.e., with the set of realized requests known in advance, denoted by BEST. (Note that BEST is not implementable in practice, as decisions have to be made immediately when requests arrive.) Recall that current practice is to accept a maximum number of deliveries per time slot. In our experiments this maximum has been set to 2 to encourage feasible sets of delivery routes.
- MAX TIME: This is the average maximum time to make an "accept" or "reject" decision over the requests in an arrival stream.
- STOPS: This is the average number of accepted deliveries among the requests in an arrival stream.
- FAIL: FAIL is the average number of accepted deliveries not visited on the final set of delivery routes. Note that not visiting an accepted delivery request can only happen when we consider current practice, i.e., SLOT.
- REV: This is the average revenue collected from the accepted requests.
- COST: COST is the average delivery costs for servicing the accepted requests.
- PROFIT: This is the average profit, i.e., the revenue collected from deliveries minus the delivery costs.

In all methods except SLOT, the decisions are based on delivery routes produced by the insertion heuristics. For all insertion heuristics, the GRASP implementation randomly chooses from among the two most profitable insertions. Given the randomization, each set of routes is built four times, and the final accept/reject decision is based on the most profitable of these runs.

For the method using an insertion criterion that tries to capture synergies between requests (REG), we use a circle surrounding the location of the request under consideration with a radius equal to five percent of the grid dimension.

In the first experiment, we use identical capacity requirements of requests (one unit), identical revenues per request ($40), and delivery costs related to distance at $1 per unit. Furthermore, we consider a single vehicle with capacity 24. We assume that each customer initially has a 24% probability of generating a delivery request. (Therefore, the expected number of realized delivery requests is 24, which equals vehicle capacity.) Each customer that requests a delivery has a time slot profile with two acceptable time slots. The first acceptable time slot is chosen randomly. The second is the one immediately following the first, unless the first acceptable time slot is the last one of the day, in which case the second acceptable time slot will be the first one of the next day. The results can be found in Table 1 for the $30 \times 30$ grid and in Table 2 for the $60 \times 60$ grid.

For the $30 \times 30$ grid, we see that dynamically testing the feasibility of an insertion, as opposed to accepting only a fixed number of requests per time slot, results in a significantly higher profit due to the increased number of deliveries now possible. Some of the profitability-based methods perform even better, but not all. However, we have to realize that with the expected demand being equal to vehicle capacity, there is little opportunity for profitability-based methods to excel. Observe too that the profitability-based methods produce profits that are fairly close to those produced with perfect information (BEST). This is due, in part, to the fact that the region is small enough so that the variation in costs between different delivery routes is relatively small. The situation changes for the $60 \times 60$ grid. As the delivery costs increase relative to the revenues, it is no longer desirable to satisfy all demand (the average number of stops is substantially less), so some profitability-based methods, particularly REG, can now do better than DYN. Note that

**Table 1** **Base Results for $30 \times 30$ Grid**

| PROB | DEC TYPE | MAX TIME | STOPS | REV | COST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|
| 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 0.24 | DYN | 0.02 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 0.24 | DSR | 0.43 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 0.24 | PATH | 9.25 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 0.24 | REG | 0.88 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 0.24 | DIFF | 0.51 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |

**Table 2** Base Results for $60 \times 60$ Grid

| PROB | DEC TYPE | MAX TIME | STOPS | REV | COST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|
| 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 0.24 | DYN | 0.02 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 0.24 | DSR | 0.31 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 0.24 | PATH | 1.63 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 0.24 | REG | 0.36 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 0.24 | DIFF | 0.36 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |

a small percentage of deliveries accepted by the SLOT method end up not being visited on the final delivery route. Finally, we observe that the computation time for the method based on computing the expected path length between two accepted requests (PATH) is the most computationally intensive. This is especially true for the $30 \times 30$ grid, since more delivery requests can be feasibly inserted on the tentative routes.

**6.2.1. Varying the Probability of Requests Being Generated.** Next we investigate the impact of increasing the probability of a request being generated before the cutoff time. Since all other instance characteristics are the same, specifically the vehicle capacity, the ratio of expected demand to vehicle capacity increases. Intuitively, if this ratio is less than one, most delivery requests should be accepted; if this ratio is greater than one, only a subset of the requests can be accepted, and selecting those requests has to be done carefully. Consequently, when the probability increases, the value of using profitability-based methods should become more pronounced. The results can be found in Table 3 for the $30 \times 30$ grid and in Table 4 for the $60 \times 60$ grid.

For the $60 \times 60$ grid, we see that the number of stops as well as the profit steadily increases as the probability of generating a request increases for the profitability-based methods, as they are all able to select a set of delivery requests with "compatible" locations and time slot profiles. On the other hand, there is little or no improvement in profit for the method that dynamically tests the feasibility of inserting an incoming request. This is not surprising because the number of accepted requests (and thus the revenue) hardly increases and no effort is made to select a set of delivery requests that can be served with low-cost delivery routes. Overall, PATH seems to have an edge over the other methods on the $60 \times 60$ grid, particularly for the higher probabilities. For the $30 \times 30$ grid, only with perfect information (BEST) is there a steady increase in profit, as delivery requests are carefully selected to minimize delivery costs. The other methods do not have the luxury of being that careful, and as the variations in delivery costs are smaller for this grid, these methods play it safe and accept any reasonably profitable request. Again, we can observe that PATH has higher computational requirements, but PATH and REG both perform well on this denser grid.

**Table 3** Results for $30 \times 30$ Grid for Different Request Probabilities

| PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|
| 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 0.24 | DYN | 0.02 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 0.24 | DSR | 0.43 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 0.24 | PATH | 9.25 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 0.24 | REG | 0.88 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 0.24 | DIFF | 0.51 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |
| 0.32 | BEST | 0.00 | 24.00 | 960.00 | 211.33 | 0.00 | 748.67 |
| 0.32 | SLOT | 0.01 | 22.50 | 900.00 | 342.02 | 0.00 | 557.98 |
| 0.32 | DYN | 0.02 | 24.00 | 960.00 | 310.54 | 0.00 | 649.46 |
| 0.32 | DSR | 0.41 | 24.00 | 960.00 | 291.71 | 0.00 | 668.29 |
| 0.32 | PATH | 9.11 | 24.00 | 960.00 | 285.79 | 0.00 | 674.22 |
| 0.32 | REG | 0.74 | 24.00 | 960.00 | 295.17 | 0.00 | 664.83 |
| 0.32 | DIFF | 0.50 | 23.60 | 944.00 | 285.70 | 0.00 | 658.30 |
| 0.40 | BEST | 0.00 | 24.00 | 960.00 | 180.81 | 0.00 | 779.19 |
| 0.40 | SLOT | 0.01 | 23.80 | 952.00 | 351.49 | 0.00 | 600.51 |
| 0.40 | DYN | 0.02 | 24.00 | 960.00 | 306.18 | 0.00 | 653.82 |
| 0.40 | DSR | 0.33 | 24.00 | 960.00 | 296.17 | 0.00 | 663.83 |
| 0.40 | PATH | 7.32 | 24.00 | 960.00 | 279.62 | 0.00 | 680.38 |
| 0.40 | REG | 0.44 | 24.00 | 960.00 | 268.13 | 0.00 | 691.87 |
| 0.40 | DIFF | 0.39 | 24.00 | 960.00 | 289.84 | 0.00 | 670.16 |
| 0.48 | BEST | 0.00 | 24.00 | 960.00 | 167.36 | 0.00 | 792.64 |
| 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 362.10 | 0.00 | 597.90 |
| 0.48 | DYN | 0.02 | 24.00 | 960.00 | 302.24 | 0.00 | 657.76 |
| 0.48 | DSR | 0.24 | 24.00 | 960.00 | 291.02 | 0.00 | 668.98 |
| 0.48 | PATH | 5.23 | 24.00 | 960.00 | 271.86 | 0.00 | 688.14 |
| 0.48 | REG | 0.35 | 24.00 | 960.00 | 272.31 | 0.00 | 687.69 |
| 0.48 | DIFF | 0.36 | 24.00 | 960.00 | 288.53 | 0.00 | 671.47 |

**6.2.2. Varying the Number of Acceptable Time Slots.** Next we examine the impact of changing the number of acceptable time slots in the time slot profile. Note that when a request is accepted, we still have to assign a single feasible time slot and guarantee that the delivery will take place during that time slot. When the number of acceptable time slots in the time slot profile increases, the number of possible feasible insertions at each iteration of the insertion heuristics is likely to increase too, and thus profits should go up. The results can be found in Table 5 for the $30 \times 30$ grid and in Table 6 for the $60 \times 60$ grid.

For the $30 \times 30$ grid, all methods except SLOT are able to exploit the extra flexibility to reduce delivery costs and thus increase profits. The number of stops on the delivery routes remains the same, but the delivery costs are reduced. Note the significant increase in computation times for PATH when the ratio of expected demand to vehicle capacity is 1 (from 4.77 seconds to 9.24 seconds to 20.77 seconds) when the number of acceptable time slots increases. This is due to the fact that more requests can feasibly be inserted on the tentative routes. For the

**Table 4** Results for $60 \times 60$ Grid for Different Request Probabilities

| PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|
| 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 0.24 | DYN | 0.02 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 0.24 | DSR | 0.31 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 0.24 | PATH | 1.63 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 0.24 | REG | 0.36 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 0.24 | DIFF | 0.36 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |
| 0.32 | BEST | 0.00 | 23.30 | 932.00 | 418.89 | 0.00 | 513.11 |
| 0.32 | SLOT | 0.02 | 21.60 | 864.00 | 622.69 | 0.90 | 241.31 |
| 0.32 | DYN | 0.02 | 24.00 | 960.00 | 605.35 | 0.00 | 354.65 |
| 0.32 | DSR | 0.28 | 23.00 | 920.00 | 476.17 | 0.00 | 443.83 |
| 0.32 | PATH | 1.85 | 21.50 | 860.00 | 421.40 | 0.00 | 438.60 |
| 0.32 | REG | 0.43 | 22.30 | 892.00 | 464.19 | 0.00 | 427.81 |
| 0.32 | DIFF | 0.32 | 18.90 | 756.00 | 361.86 | 0.00 | 394.14 |
| 0.40 | BEST | 0.01 | 24.00 | 960.00 | 373.61 | 0.00 | 586.39 |
| 0.40 | SLOT | 0.05 | 22.70 | 908.00 | 645.62 | 1.10 | 262.38 |
| 0.40 | DYN | 0.05 | 24.00 | 960.00 | 609.58 | 0.00 | 350.42 |
| 0.40 | DSR | 0.46 | 24.00 | 960.00 | 483.79 | 0.00 | 476.21 |
| 0.40 | PATH | 2.20 | 23.20 | 928.00 | 432.79 | 0.00 | 495.21 |
| 0.40 | REG | 0.45 | 23.90 | 956.00 | 494.06 | 0.00 | 461.94 |
| 0.40 | DIFF | 0.30 | 22.00 | 880.00 | 403.39 | 0.00 | 476.61 |
| 0.48 | BEST | 0.00 | 23.90 | 956.00 | 341.26 | 0.00 | 614.74 |
| 0.48 | SLOT | 0.01 | 22.50 | 900.00 | 627.46 | 1.50 | 272.54 |
| 0.48 | DYN | 0.02 | 24.00 | 960.00 | 634.94 | 0.00 | 325.06 |
| 0.48 | DSR | 0.31 | 24.00 | 960.00 | 466.32 | 0.00 | 493.68 |
| 0.48 | PATH | 1.73 | 23.20 | 928.00 | 410.55 | 0.00 | 517.45 |
| 0.48 | REG | 0.41 | 24.00 | 960.00 | 486.41 | 0.00 | 473.59 |
| 0.48 | DIFF | 0.30 | 23.40 | 936.00 | 443.49 | 0.00 | 492.51 |

$60 \times 60$ grid and a ratio of expected demand to vehicle capacity equal to 1, all methods exploit the extra flexibility to increase the number of accepted deliveries (e.g., PATH goes from 14.7 to 16.2 to 18.4). Since this can be done with a relatively small increase in delivery cost, the profit goes up. When the ratio of expected demand to vehicle capacity equals 2, all methods except SLOT exploit the extra flexibility to reduce the delivery costs (e.g., PATH goes from 436.07 to 410.55 to 375.71).

**6.2.3. Varying the Number of Vehicles.** Next we study the impact of distributing capacity over one, two, and three vehicles. The results can be found in Table 7 for the $30 \times 30$ grid and in Table 8 for the $60 \times 60$ grid.

It is interesting to see that keeping the capacity the same but spreading it over multiple vehicles allows for an increase in the total revenue. At first, this may seem counterintuitive, because it is cheaper (distance wise) to visit $k$ customers on a single route than it is to visit the same $k$ customers on two or more routes. However, there are two reasons for the observed behavior. First, the number of stops may increase when the capacity is distributed over multiple vehicles, because it may have been time that was constraining the number of stops with a single vehicle and not the vehicle capacity. Second, the delivery costs may decrease because more customers may be visited during more convenient time slots. With multiple vehicles there are more opportunities to visit customers in a given time slot. The improvement can only be seen by BEST on the $30 \times 30$ grid but is realized by the probability based methods on the $60 \times 60$ grid.

**Table 5** Results for $30 \times 30$ Grid for Different Numbers of Acceptable Time Slots

| # SLOTS | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.24 | BEST | 0.00 | 23.70 | 948.00 | 299.53 | 0.00 | 648.47 |
| 1 | 0.24 | SLOT | 0.01 | 18.20 | 728.00 | 282.12 | 0.00 | 445.88 |
| 1 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 326.99 | 0.00 | 629.01 |
| 1 | 0.24 | DSR | 0.22 | 23.90 | 956.00 | 323.28 | 0.00 | 632.72 |
| 1 | 0.24 | PATH | 4.77 | 23.50 | 940.00 | 307.01 | 0.00 | 632.99 |
| 1 | 0.24 | REG | 0.32 | 23.90 | 956.00 | 324.20 | 0.00 | 631.80 |
| 1 | 0.24 | DIFF | 0.26 | 22.30 | 892.00 | 290.39 | 0.00 | 601.62 |
| 2 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 2 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 2 | 0.24 | DYN | 0.01 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 2 | 0.24 | DSR | 0.45 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 2 | 0.24 | PATH | 9.24 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 2 | 0.24 | REG | 0.58 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 2 | 0.24 | DIFF | 0.49 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |
| 4 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 218.79 | 0.00 | 737.21 |
| 4 | 0.24 | SLOT | 0.01 | 22.00 | 880.00 | 332.51 | 0.00 | 547.49 |
| 4 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 252.76 | 0.00 | 703.24 |
| 4 | 0.24 | DSR | 0.84 | 23.90 | 956.00 | 249.63 | 0.00 | 706.37 |
| 4 | 0.24 | PATH | 20.77 | 23.70 | 948.00 | 229.11 | 0.00 | 718.89 |
| 4 | 0.24 | REG | 1.17 | 23.80 | 952.00 | 242.12 | 0.00 | 709.88 |
| 4 | 0.24 | DIFF | 1.00 | 22.60 | 904.00 | 228.34 | 0.00 | 675.66 |
| 1 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 185.57 | 0.00 | 774.43 |
| 1 | 0.48 | SLOT | 0.01 | 22.30 | 892.00 | 329.77 | 0.00 | 562.23 |
| 1 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 344.72 | 0.00 | 615.29 |
| 1 | 0.48 | DSR | 0.13 | 24.00 | 960.00 | 310.35 | 0.00 | 649.65 |
| 1 | 0.48 | PATH | 2.94 | 24.00 | 960.00 | 305.19 | 0.00 | 654.81 |
| 1 | 0.48 | REG | 0.19 | 24.00 | 960.00 | 310.09 | 0.00 | 649.91 |
| 1 | 0.48 | DIFF | 0.16 | 24.00 | 960.00 | 311.39 | 0.00 | 648.61 |
| 2 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 167.36 | 0.00 | 792.64 |
| 2 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 362.10 | 0.00 | 597.90 |
| 2 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 302.24 | 0.00 | 657.76 |
| 2 | 0.48 | DSR | 0.23 | 24.00 | 960.00 | 291.02 | 0.00 | 668.98 |
| 2 | 0.48 | PATH | 5.21 | 24.00 | 960.00 | 271.86 | 0.00 | 688.14 |
| 2 | 0.48 | REG | 0.36 | 24.00 | 960.00 | 272.31 | 0.00 | 687.69 |
| 2 | 0.48 | DIFF | 0.29 | 24.00 | 960.00 | 288.53 | 0.00 | 671.47 |
| 4 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 127.75 | 0.00 | 832.25 |
| 4 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 363.02 | 0.00 | 596.98 |
| 4 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 263.30 | 0.00 | 696.70 |
| 4 | 0.48 | DSR | 0.44 | 24.00 | 960.00 | 240.25 | 0.00 | 719.75 |
| 4 | 0.48 | PATH | 8.83 | 24.00 | 960.00 | 233.53 | 0.00 | 726.47 |
| 4 | 0.48 | REG | 0.63 | 24.00 | 960.00 | 233.58 | 0.00 | 726.42 |
| 4 | 0.48 | DIFF | 0.53 | 24.00 | 960.00 | 242.34 | 0.00 | 717.67 |

**6.2.4. Varying the Revenue per Delivery Request.** In all the experiments discussed up to now, each realized and accepted delivery request produced \$40 in revenue. To see what happens when revenues per delivery vary, we conducted an experiment in which the customers are divided into two sets. For half of the customers, a realized delivery request results in

**Table 6** Results for $60 \times 60$ Grid for Different Numbers of Acceptable Time Slots

| # SLOTS | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.24 | BEST | 0.00 | 19.40 | 776.00 | 423.28 | 0.00 | 352.72 |
| 1 | 0.24 | SLOT | 0.01 | 17.90 | 716.00 | 549.58 | 0.30 | 166.42 |
| 1 | 0.24 | DYN | 0.01 | 21.10 | 844.00 | 585.79 | 0.00 | 258.21 |
| 1 | 0.24 | DSR | 0.11 | 17.10 | 684.00 | 388.28 | 0.00 | 295.72 |
| 1 | 0.24 | PATH | 0.77 | 14.70 | 588.00 | 303.67 | 0.00 | 284.33 |
| 1 | 0.24 | REG | 0.15 | 17.60 | 704.00 | 399.88 | 0.00 | 304.12 |
| 1 | 0.24 | DIFF | 0.12 | 13.00 | 520.00 | 265.80 | 0.00 | 254.20 |
| 2 | 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 2 | 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 2 | 0.24 | DYN | 0.01 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 2 | 0.24 | DSR | 0.25 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 2 | 0.24 | PATH | 1.50 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 2 | 0.24 | REG | 0.32 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 2 | 0.24 | DIFF | 0.24 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |
| 4 | 0.24 | BEST | 0.00 | 22.60 | 904.00 | 410.50 | 0.00 | 493.50 |
| 4 | 0.24 | SLOT | 0.01 | 20.90 | 836.00 | 598.22 | 1.10 | 237.78 |
| 4 | 0.24 | DYN | 0.01 | 23.80 | 952.00 | 528.25 | 0.00 | 423.75 |
| 4 | 0.24 | DSR | 0.61 | 21.30 | 852.00 | 405.99 | 0.00 | 446.01 |
| 4 | 0.24 | PATH | 4.06 | 18.40 | 736.00 | 336.75 | 0.00 | 399.25 |
| 4 | 0.24 | REG | 0.73 | 21.70 | 868.00 | 419.80 | 0.00 | 448.20 |
| 4 | 0.24 | DIFF | 0.55 | 14.60 | 584.00 | 248.64 | 0.00 | 335.36 |
| 1 | 0.48 | BEST | 0.00 | 23.80 | 952.00 | 386.03 | 0.00 | 565.97 |
| 1 | 0.48 | SLOT | 0.01 | 21.50 | 860.00 | 615.23 | 0.80 | 244.77 |
| 1 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 642.04 | 0.00 | 317.96 |
| 1 | 0.48 | DSR | 0.11 | 24.00 | 960.00 | 502.37 | 0.00 | 457.63 |
| 1 | 0.48 | PATH | 0.80 | 23.10 | 924.00 | 436.07 | 0.00 | 487.93 |
| 1 | 0.48 | REG | 0.14 | 24.00 | 960.00 | 512.00 | 0.00 | 448.00 |
| 1 | 0.48 | DIFF | 0.12 | 22.90 | 916.00 | 436.55 | 0.00 | 479.45 |
| 2 | 0.48 | BEST | 0.00 | 23.90 | 956.00 | 341.26 | 0.00 | 614.74 |
| 2 | 0.48 | SLOT | 0.01 | 22.50 | 900.00 | 627.46 | 1.50 | 272.54 |
| 2 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 634.94 | 0.00 | 325.06 |
| 2 | 0.48 | DSR | 0.22 | 24.00 | 960.00 | 466.32 | 0.00 | 493.68 |
| 2 | 0.48 | PATH | 1.67 | 23.20 | 928.00 | 410.55 | 0.00 | 517.45 |
| 2 | 0.48 | REG | 0.28 | 24.00 | 960.00 | 486.41 | 0.00 | 473.59 |
| 2 | 0.48 | DIFF | 0.23 | 23.40 | 936.00 | 443.49 | 0.00 | 492.51 |
| 4 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 266.78 | 0.00 | 693.22 |
| 4 | 0.48 | SLOT | 0.01 | 22.80 | 912.00 | 654.39 | 1.20 | 257.61 |
| 4 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 563.90 | 0.00 | 396.10 |
| 4 | 0.48 | DSR | 0.42 | 24.00 | 960.00 | 418.11 | 0.00 | 541.89 |
| 4 | 0.48 | PATH | 3.80 | 24.00 | 960.00 | 375.71 | 0.00 | 584.29 |
| 4 | 0.48 | REG | 0.55 | 24.00 | 960.00 | 393.57 | 0.00 | 566.43 |
| 4 | 0.48 | DIFF | 0.47 | 23.60 | 944.00 | 380.66 | 0.00 | 563.34 |

**Table 7** Results for $30 \times 30$ Grid for Different Numbers of Vehicles

| TRKS | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 1 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 1 | 0.24 | DYN | 0.01 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 1 | 0.24 | DSR | 0.42 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 1 | 0.24 | PATH | 9.23 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 1 | 0.24 | REG | 0.57 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 1 | 0.24 | DIFF | 0.49 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |
| 2 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 255.57 | 0.00 | 700.43 |
| 2 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 264.39 | 0.00 | 551.61 |
| 2 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 280.81 | 0.00 | 675.19 |
| 2 | 0.24 | DSR | 0.46 | 23.90 | 956.00 | 283.86 | 0.00 | 672.14 |
| 2 | 0.24 | PATH | 7.39 | 23.60 | 944.00 | 272.11 | 0.00 | 671.89 |
| 2 | 0.24 | REG | 0.71 | 23.70 | 948.00 | 279.56 | 0.00 | 668.44 |
| 2 | 0.24 | DIFF | 0.54 | 22.10 | 884.00 | 252.82 | 0.00 | 631.17 |
| 3 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 250.02 | 0.00 | 705.98 |
| 3 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 269.33 | 0.00 | 546.67 |
| 3 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 287.78 | 0.00 | 668.22 |
| 3 | 0.24 | DSR | 0.46 | 23.90 | 956.00 | 290.59 | 0.00 | 665.41 |
| 3 | 0.24 | PATH | 5.44 | 23.50 | 940.00 | 275.29 | 0.00 | 664.71 |
| 3 | 0.24 | REG | 0.75 | 23.80 | 952.00 | 285.96 | 0.00 | 666.04 |
| 3 | 0.24 | DIFF | 0.53 | 22.00 | 880.00 | 259.04 | 0.00 | 620.96 |
| 1 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 167.36 | 0.00 | 792.64 |
| 1 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 362.10 | 0.00 | 597.90 |
| 1 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 302.24 | 0.00 | 657.76 |
| 1 | 0.48 | DSR | 0.23 | 24.00 | 960.00 | 291.02 | 0.00 | 668.98 |
| 1 | 0.48 | PATH | 5.24 | 24.00 | 960.00 | 271.86 | 0.00 | 688.14 |
| 1 | 0.48 | REG | 0.34 | 24.00 | 960.00 | 272.31 | 0.00 | 687.69 |
| 1 | 0.48 | DIFF | 0.28 | 24.00 | 960.00 | 288.53 | 0.00 | 671.47 |
| 2 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 151.13 | 0.00 | 808.88 |
| 2 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 313.04 | 0.00 | 646.96 |
| 2 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 289.64 | 0.00 | 670.36 |
| 2 | 0.48 | DSR | 0.24 | 24.00 | 960.00 | 273.11 | 0.00 | 686.89 |
| 2 | 0.48 | PATH | 2.16 | 24.00 | 960.00 | 273.77 | 0.00 | 686.23 |
| 2 | 0.48 | REG | 0.40 | 24.00 | 960.00 | 267.69 | 0.00 | 692.31 |
| 2 | 0.48 | DIFF | 0.27 | 24.00 | 960.00 | 264.65 | 0.00 | 695.35 |
| 3 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 155.41 | 0.00 | 804.59 |
| 3 | 0.48 | SLOT | 0.02 | 24.00 | 960.00 | 290.07 | 0.00 | 669.93 |
| 3 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 288.32 | 0.00 | 671.68 |
| 3 | 0.48 | DSR | 0.24 | 24.00 | 960.00 | 277.07 | 0.00 | 682.92 |
| 3 | 0.48 | PATH | 1.29 | 24.00 | 960.00 | 267.45 | 0.00 | 692.55 |
| 3 | 0.48 | REG | 0.41 | 24.00 | 960.00 | 264.46 | 0.00 | 695.54 |
| 3 | 0.48 | DIFF | 0.27 | 24.00 | 960.00 | 272.95 | 0.00 | 687.05 |

a low revenue, and for the other half, a realized delivery request results in a high revenue. The results can be found in Table 9 for the $30 \times 30$ grid and in Table 10 for the $60 \times 60$ grid.

It is clear that when requests have different revenues and the objective is to maximize profit, it is important to select and accept high-revenue requests. However, when the ratio of expected demand to vehicle capacity is equal to 1 and almost all delivery requests can and should be accepted, there is little room for improvement. This can be seen in the results for the $30 \times 30$ grid, where there is little difference in performance between the profitability-based methods and DYN. The situation changes when the ratio of expected demand to vehicle capacity is equal to 2. Especially for the $30 \times 30$ grid, we see that with perfect information significantly higher revenues and thus profits can be realized (revenues increase from 960 to 1,334 to 1,840). Even though profit-based methods do a little better than SLOT and DYN in recognizing these opportunities, they do not get close to what can be done with perfect information. This is due to the fact that as we get closer to the cutoff time and the probabilities of requests still being realized get smaller (recall that the probability of a request being realized decreases linearly over time), the profitability-based methods are not taking any chances and are accepting low-revenue requests

**Table 8** Results for $60 \times 60$ Grid for Different Numbers of Vehicles

| TRKS | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 1 | 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 1 | 0.24 | DYN | 0.02 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 1 | 0.24 | DSR | 0.25 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 1 | 0.24 | PATH | 1.50 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 1 | 0.24 | REG | 0.32 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 1 | 0.24 | DIFF | 0.24 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |
| 2 | 0.24 | BEST | 0.00 | 22.20 | 888.00 | 427.16 | 0.00 | 460.84 |
| 2 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 530.07 | 0.00 | 285.93 |
| 2 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 561.74 | 0.00 | 394.26 |
| 2 | 0.24 | DSR | 0.42 | 21.50 | 860.00 | 493.24 | 0.00 | 366.76 |
| 2 | 0.24 | PATH | 2.75 | 21.30 | 852.00 | 456.76 | 0.00 | 395.24 |
| 2 | 0.24 | REG | 0.57 | 21.20 | 848.00 | 437.03 | 0.00 | 410.97 |
| 2 | 0.24 | DIFF | 0.43 | 14.20 | 568.00 | 281.20 | 0.00 | 286.80 |
| 3 | 0.24 | BEST | 0.00 | 22.90 | 916.00 | 448.94 | 0.00 | 467.06 |
| 3 | 0.24 | SLOT | 0.01 | 20.30 | 812.00 | 535.66 | 0.10 | 276.34 |
| 3 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 567.64 | 0.00 | 388.36 |
| 3 | 0.24 | DSR | 0.44 | 22.00 | 880.00 | 502.50 | 0.00 | 377.50 |
| 3 | 0.24 | PATH | 2.43 | 21.60 | 864.00 | 499.21 | 0.00 | 364.79 |
| 3 | 0.24 | REG | 0.67 | 20.80 | 832.00 | 485.67 | 0.00 | 346.33 |
| 3 | 0.24 | DIFF | 0.49 | 14.00 | 560.00 | 273.41 | 0.00 | 286.59 |
| 1 | 0.48 | BEST | 0.00 | 23.90 | 956.00 | 341.26 | 0.00 | 614.74 |
| 1 | 0.48 | SLOT | 0.01 | 22.50 | 900.00 | 627.46 | 1.50 | 272.54 |
| 1 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 634.94 | 0.00 | 325.06 |
| 1 | 0.48 | DSR | 0.22 | 24.00 | 960.00 | 466.32 | 0.00 | 493.68 |
| 1 | 0.48 | PATH | 1.67 | 23.20 | 928.00 | 410.55 | 0.00 | 517.45 |
| 1 | 0.48 | REG | 0.28 | 24.00 | 960.00 | 486.41 | 0.00 | 473.59 |
| 1 | 0.48 | DIFF | 0.23 | 23.40 | 936.00 | 443.49 | 0.00 | 492.51 |
| 2 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 306.90 | 0.00 | 653.10 |
| 2 | 0.48 | SLOT | 0.01 | 23.90 | 956.00 | 632.79 | 0.10 | 323.21 |
| 2 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 604.98 | 0.00 | 355.02 |
| 2 | 0.48 | DSR | 0.24 | 24.00 | 960.00 | 501.47 | 0.00 | 458.53 |
| 2 | 0.48 | PATH | 1.43 | 24.00 | 960.00 | 468.21 | 0.00 | 491.79 |
| 2 | 0.48 | REG | 0.36 | 24.00 | 960.00 | 475.17 | 0.00 | 484.83 |
| 2 | 0.48 | DIFF | 0.27 | 22.10 | 884.00 | 383.70 | 0.00 | 500.30 |
| 3 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 312.56 | 0.00 | 647.44 |
| 3 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 580.14 | 0.00 | 379.86 |
| 3 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 590.04 | 0.00 | 369.96 |
| 3 | 0.48 | DSR | 0.24 | 24.00 | 960.00 | 496.57 | 0.00 | 463.43 |
| 3 | 0.48 | PATH | 1.01 | 24.00 | 960.00 | 450.80 | 0.00 | 509.20 |
| 3 | 0.48 | REG | 0.39 | 24.00 | 960.00 | 468.34 | 0.00 | 491.66 |
| 3 | 0.48 | DIFF | 0.28 | 23.30 | 932.00 | 432.55 | 0.00 | 499.45 |

that have materialized rather than waiting for high-revenue requests that have a small chance of occurring (thus low expected revenue) with similar distances. For the $60 \times 60$ grid, where the costs between customers are more varied, the profitability-based methods do better with higher probabilities (higher expected revenue), in the sense that they clearly outperform SLOT and DYN and get closer to what can be achieved with perfect information.

**6.2.5. Varying the Time Slot Width.** Finally, we experiment with different widths of the time slots. In our definition of the HDP, we stated that there are nonoverlapping time slots of one hour, and in the computational experiments so far, we have used

**Table 9** Results for $30 \times 30$ Grid with Different Revenues per Request

| LOW REV | UPPER REV | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|---|
| 40.00 | 40.00 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 40.00 | 40.00 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 40.00 | 40.00 | 0.24 | DYN | 0.01 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 40.00 | 40.00 | 0.24 | DSR | 0.42 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 40.00 | 40.00 | 0.24 | PATH | 9.23 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 40.00 | 40.00 | 0.24 | REG | 0.58 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 40.00 | 40.00 | 0.24 | DIFF | 0.49 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |
| 40.00 | 60.00 | 0.24 | BEST | 0.00 | 23.80 | 1,192.00 | 264.06 | 0.00 | 927.94 |
| 40.00 | 60.00 | 0.24 | SLOT | 0.01 | 20.40 | 1,024.00 | 313.37 | 0.00 | 710.63 |
| 40.00 | 60.00 | 0.24 | DYN | 0.02 | 23.90 | 1,188.00 | 292.55 | 0.00 | 895.45 |
| 40.00 | 60.00 | 0.24 | DSR | 0.42 | 23.90 | 1,190.00 | 291.21 | 0.00 | 898.79 |
| 40.00 | 60.00 | 0.24 | PATH | 7.96 | 23.80 | 1,186.00 | 281.21 | 0.00 | 904.79 |
| 40.00 | 60.00 | 0.24 | REG | 0.57 | 23.90 | 1,192.00 | 288.89 | 0.00 | 903.11 |
| 40.00 | 60.00 | 0.24 | DIFF | 0.49 | 23.40 | 1,172.00 | 286.33 | 0.00 | 885.67 |
| 40.00 | 80.00 | 0.24 | BEST | 0.00 | 23.80 | 1,432.00 | 268.14 | 0.00 | 1,163.86 |
| 40.00 | 80.00 | 0.24 | SLOT | 0.01 | 20.40 | 1,232.00 | 310.53 | 0.00 | 921.47 |
| 40.00 | 80.00 | 0.24 | DYN | 0.02 | 23.90 | 1,420.00 | 291.80 | 0.00 | 1,128.20 |
| 40.00 | 80.00 | 0.24 | DSR | 0.42 | 23.90 | 1,420.00 | 289.98 | 0.00 | 1,130.02 |
| 40.00 | 80.00 | 0.24 | PATH | 7.70 | 23.80 | 1,420.00 | 281.11 | 0.00 | 1,138.90 |
| 40.00 | 80.00 | 0.24 | REG | 0.58 | 23.90 | 1,424.00 | 284.14 | 0.00 | 1,139.86 |
| 40.00 | 80.00 | 0.24 | DIFF | 0.49 | 23.40 | 1,408.00 | 272.08 | 0.00 | 1,135.92 |
| 40.00 | 40.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 167.36 | 0.00 | 792.64 |
| 40.00 | 40.00 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 362.10 | 0.00 | 597.90 |
| 40.00 | 40.00 | 0.48 | DYN | 0.01 | 24.00 | 960.00 | 302.24 | 0.00 | 657.76 |
| 40.00 | 40.00 | 0.48 | DSR | 0.23 | 24.00 | 960.00 | 291.02 | 0.00 | 668.98 |
| 40.00 | 40.00 | 0.48 | PATH | 5.25 | 24.00 | 960.00 | 271.86 | 0.00 | 688.14 |
| 40.00 | 40.00 | 0.48 | REG | 0.34 | 24.00 | 960.00 | 272.31 | 0.00 | 687.69 |
| 40.00 | 40.00 | 0.48 | DIFF | 0.28 | 24.00 | 960.00 | 288.53 | 0.00 | 671.47 |
| 40.00 | 60.00 | 0.48 | BEST | 0.00 | 24.00 | 1,334.00 | 202.93 | 0.00 | 1,131.07 |
| 40.00 | 60.00 | 0.48 | SLOT | 0.01 | 24.00 | 1,202.00 | 348.33 | 0.00 | 853.67 |
| 40.00 | 60.00 | 0.48 | DYN | 0.02 | 24.00 | 1,188.00 | 302.84 | 0.00 | 885.16 |
| 40.00 | 60.00 | 0.48 | DSR | 0.23 | 24.00 | 1,208.00 | 295.13 | 0.00 | 912.88 |
| 40.00 | 60.00 | 0.48 | PATH | 4.61 | 24.00 | 1,204.00 | 293.40 | 0.00 | 910.60 |
| 40.00 | 60.00 | 0.48 | REG | 0.35 | 24.00 | 1,218.00 | 276.26 | 0.00 | 941.74 |
| 40.00 | 60.00 | 0.48 | DIFF | 0.28 | 24.00 | 1,206.00 | 291.94 | 0.00 | 914.06 |
| 40.00 | 80.00 | 0.48 | BEST | 0.00 | 24.00 | 1,840.00 | 276.02 | 0.00 | 1,563.98 |
| 40.00 | 80.00 | 0.48 | SLOT | 0.01 | 24.00 | 1,444.00 | 351.35 | 0.00 | 1,092.65 |
| 40.00 | 80.00 | 0.48 | DYN | 0.02 | 24.00 | 1,416.00 | 298.26 | 0.00 | 1,117.74 |
| 40.00 | 80.00 | 0.48 | DSR | 0.23 | 24.00 | 1,468.00 | 277.07 | 0.00 | 1,190.93 |
| 40.00 | 80.00 | 0.48 | PATH | 4.33 | 24.00 | 1,488.00 | 278.09 | 0.00 | 1,209.91 |
| 40.00 | 80.00 | 0.48 | REG | 0.35 | 24.00 | 1,516.00 | 281.17 | 0.00 | 1,234.83 |
| 40.00 | 80.00 | 0.48 | DIFF | 0.28 | 24.00 | 1,472.00 | 279.16 | 0.00 | 1,192.84 |

12 time slots of one hour. However, the width of these time slots is a strategic decision in the design of a HDS, and it is interesting to study the impact the time slot width has on the profitability of the home delivery operation. Note that in practice the marketing department is highly involved in choosing the time slot width to gain market share from the competition, so this decision is often based on how time slot width will affect demand as well as the resulting costs. As before, each customer has two acceptable time slots in his or her time slot profile. We adjust the number of requests allowed per time slot in the SLOT method so as to allow two requests per hour. Thus, if the time slot width is two hours, four delivery requests can be accepted in each slot. We have experimented with widths of 30, 60, 120, and 180 minutes. The results

**Table 10** Results for $60 \times 60$ Grid with Different Revenues per Request

| LOW REV | UPPER REV | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|---|
| 40.00 | 40.00 | 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 40.00 | 40.00 | 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 40.00 | 40.00 | 0.24 | DYN | 0.02 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 40.00 | 40.00 | 0.24 | DSR | 0.25 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 40.00 | 40.00 | 0.24 | PATH | 1.50 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 40.00 | 40.00 | 0.24 | REG | 0.31 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 40.00 | 40.00 | 0.24 | DIFF | 0.24 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |
| 40.00 | 60.00 | 0.24 | BEST | 0.00 | 21.90 | 1,102.00 | 455.49 | 0.00 | 646.51 |
| 40.00 | 60.00 | 0.24 | SLOT | 0.01 | 19.50 | 984.00 | 572.25 | 0.90 | 411.75 |
| 40.00 | 60.00 | 0.24 | DYN | 0.01 | 22.90 | 1,144.00 | 576.75 | 0.00 | 567.25 |
| 40.00 | 60.00 | 0.24 | DSR | 0.25 | 20.10 | 1,018.00 | 436.45 | 0.00 | 581.55 |
| 40.00 | 60.00 | 0.24 | PATH | 1.48 | 19.10 | 966.00 | 415.19 | 0.00 | 550.81 |
| 40.00 | 60.00 | 0.24 | REG | 0.32 | 20.50 | 1,038.00 | 466.24 | 0.00 | 571.76 |
| 40.00 | 60.00 | 0.24 | DIFF | 0.24 | 15.80 | 814.00 | 309.88 | 0.00 | 504.12 |
| 40.00 | 80.00 | 0.24 | BEST | 0.00 | 21.60 | 1,336.00 | 467.48 | 0.00 | 868.52 |
| 40.00 | 80.00 | 0.24 | SLOT | 0.01 | 19.50 | 1,196.00 | 576.95 | 0.90 | 619.05 |
| 40.00 | 80.00 | 0.24 | DYN | 0.02 | 23.30 | 1,392.00 | 589.71 | 0.00 | 802.29 |
| 40.00 | 80.00 | 0.24 | DSR | 0.25 | 20.80 | 1,288.00 | 498.81 | 0.00 | 789.19 |
| 40.00 | 80.00 | 0.24 | PATH | 1.15 | 19.30 | 1,212.00 | 443.22 | 0.00 | 768.78 |
| 40.00 | 80.00 | 0.24 | REG | 0.32 | 21.20 | 1,308.00 | 486.81 | 0.00 | 821.20 |
| 40.00 | 80.00 | 0.24 | DIFF | 0.25 | 18.30 | 1,152.00 | 402.42 | 0.00 | 749.58 |
| 40.00 | 40.00 | 0.48 | BEST | 0.00 | 23.90 | 956.00 | 341.26 | 0.00 | 614.74 |
| 40.00 | 40.00 | 0.48 | SLOT | 0.01 | 22.50 | 900.00 | 627.46 | 1.50 | 272.54 |
| 40.00 | 40.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 634.94 | 0.00 | 325.06 |
| 40.00 | 40.00 | 0.48 | DSR | 0.22 | 24.00 | 960.00 | 466.32 | 0.00 | 493.68 |
| 40.00 | 40.00 | 0.48 | PATH | 1.66 | 23.20 | 928.00 | 410.55 | 0.00 | 517.45 |
| 40.00 | 40.00 | 0.48 | REG | 0.28 | 24.00 | 960.00 | 486.41 | 0.00 | 473.59 |
| 40.00 | 40.00 | 0.48 | DIFF | 0.23 | 23.40 | 936.00 | 443.49 | 0.00 | 492.51 |
| 40.00 | 60.00 | 0.48 | BEST | 0.00 | 24.00 | 1,256.00 | 342.79 | 0.00 | 913.21 |
| 40.00 | 60.00 | 0.48 | SLOT | 0.01 | 22.50 | 1,130.00 | 628.73 | 1.50 | 501.27 |
| 40.00 | 60.00 | 0.48 | DYN | 0.02 | 24.00 | 1,184.00 | 627.33 | 0.00 | 556.67 |
| 40.00 | 60.00 | 0.48 | DSR | 0.22 | 24.00 | 1,222.00 | 489.77 | 0.00 | 732.23 |
| 40.00 | 60.00 | 0.48 | PATH | 1.59 | 24.00 | 1,230.00 | 434.72 | 0.00 | 795.28 |
| 40.00 | 60.00 | 0.48 | REG | 0.28 | 24.00 | 1,216.00 | 474.42 | 0.00 | 741.58 |
| 40.00 | 60.00 | 0.48 | DIFF | 0.23 | 23.40 | 1,180.00 | 456.28 | 0.00 | 723.72 |
| 40.00 | 80.00 | 0.48 | BEST | 0.00 | 24.00 | 1,688.00 | 407.39 | 0.00 | 1,280.61 |
| 40.00 | 80.00 | 0.48 | SLOT | 0.01 | 22.80 | 1,376.00 | 631.45 | 1.20 | 744.55 |
| 40.00 | 80.00 | 0.48 | DYN | 0.02 | 24.00 | 1,400.00 | 640.09 | 0.00 | 759.91 |
| 40.00 | 80.00 | 0.48 | DSR | 0.22 | 24.00 | 1,520.00 | 535.26 | 0.00 | 984.74 |
| 40.00 | 80.00 | 0.48 | PATH | 1.37 | 24.00 | 1,532.00 | 485.25 | 0.00 | 1,046.75 |
| 40.00 | 80.00 | 0.48 | REG | 0.28 | 24.00 | 1,528.00 | 497.04 | 0.00 | 1,030.96 |
| 40.00 | 80.00 | 0.48 | DIFF | 0.24 | 23.90 | 1,512.00 | 507.38 | 0.00 | 1,004.62 |

can be found in Table 11 for the $30 \times 30$ grid and in Table 12 for the $60 \times 60$ grid.

We see that increasing window widths translates in steadily increasing profits. Consider, for example, method REG for the $30 \times 30$ grid with expected demand to capacity ratio of 1. The number of accepted requests is roughly the same for each time slot width, but the delivery costs go down dramatically, from 326.63 to 287.08 to 244.20 to 201.17. This clearly demonstrates that even in a region with a relatively high customer density, which is the case for the $30 \times 30$ grid, the profit is significantly affected by committing to a tight time slot for actual deliveries. Note also that with increasing window width, the number of feasible insertions becomes larger so the required computation time substantially increases, most notably for the PATH method

**Table 11** Results for $30 \times 30$ Grid for Different Time Slot Widths

| WIDTH | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 30.00 | 0.24 | BEST | 0.00 | 23.60 | 944.00 | 308.03 | 0.00 | 635.97 |
| 30.00 | 0.24 | SLOT | 0.01 | 18.60 | 744.00 | 308.68 | 0.00 | 435.32 |
| 30.00 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 336.99 | 0.00 | 619.01 |
| 30.00 | 0.24 | DSR | 0.36 | 23.70 | 948.00 | 333.97 | 0.00 | 614.03 |
| 30.00 | 0.24 | PATH | 3.26 | 23.50 | 940.00 | 329.52 | 0.00 | 610.48 |
| 30.00 | 0.24 | REG | 0.44 | 23.50 | 940.00 | 326.63 | 0.00 | 613.37 |
| 30.00 | 0.24 | DIFF | 0.38 | 22.40 | 896.00 | 302.40 | 0.00 | 593.60 |
| 60.00 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 273.01 | 0.00 | 682.99 |
| 60.00 | 0.24 | SLOT | 0.01 | 20.40 | 816.00 | 305.76 | 0.00 | 510.24 |
| 60.00 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 296.72 | 0.00 | 659.27 |
| 60.00 | 0.24 | DSR | 0.44 | 23.90 | 956.00 | 285.07 | 0.00 | 670.93 |
| 60.00 | 0.24 | PATH | 9.18 | 23.70 | 948.00 | 281.33 | 0.00 | 666.67 |
| 60.00 | 0.24 | REG | 0.60 | 23.90 | 956.00 | 287.08 | 0.00 | 668.92 |
| 60.00 | 0.24 | DIFF | 0.51 | 22.70 | 908.00 | 265.74 | 0.00 | 642.26 |
| 120.00 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 227.64 | 0.00 | 728.36 |
| 120.00 | 0.24 | SLOT | 0.01 | 22.10 | 884.00 | 277.30 | 0.00 | 606.70 |
| 120.00 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 244.28 | 0.00 | 711.72 |
| 120.00 | 0.24 | DSR | 0.49 | 23.90 | 956.00 | 239.68 | 0.00 | 716.32 |
| 120.00 | 0.24 | PATH | 29.76 | 23.80 | 952.00 | 246.82 | 0.00 | 705.18 |
| 120.00 | 0.24 | REG | 0.96 | 23.90 | 956.00 | 244.20 | 0.00 | 711.80 |
| 120.00 | 0.24 | DIFF | 0.77 | 22.90 | 916.00 | 237.33 | 0.00 | 678.67 |
| 180.00 | 0.24 | BEST | 0.00 | 23.90 | 956.00 | 189.35 | 0.00 | 766.65 |
| 180.00 | 0.24 | SLOT | 0.01 | 21.90 | 876.00 | 253.51 | 0.00 | 622.49 |
| 180.00 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 202.98 | 0.00 | 753.02 |
| 180.00 | 0.24 | DSR | 0.56 | 23.70 | 948.00 | 194.89 | 0.00 | 753.11 |
| 180.00 | 0.24 | PATH | 50.70 | 23.80 | 952.00 | 196.92 | 0.00 | 755.08 |
| 180.00 | 0.24 | REG | 1.24 | 23.80 | 952.00 | 201.17 | 0.00 | 750.83 |
| 180.00 | 0.24 | DIFF | 1.01 | 22.90 | 916.00 | 193.55 | 0.00 | 722.45 |
| 30.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 195.50 | 0.00 | 764.50 |
| 30.00 | 0.48 | SLOT | 0.01 | 23.00 | 920.00 | 370.28 | 0.00 | 549.72 |
| 30.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 352.61 | 0.00 | 607.39 |
| 30.00 | 0.48 | DSR | 0.25 | 24.00 | 960.00 | 322.21 | 0.00 | 637.79 |
| 30.00 | 0.48 | PATH | 2.51 | 24.00 | 960.00 | 307.10 | 0.00 | 652.90 |
| 30.00 | 0.48 | REG | 0.30 | 24.00 | 960.00 | 302.08 | 0.00 | 657.92 |
| 30.00 | 0.48 | DIFF | 0.26 | 24.00 | 960.00 | 336.12 | 0.00 | 623.88 |
| 60.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 167.36 | 0.00 | 792.64 |
| 60.00 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 362.10 | 0.00 | 597.90 |
| 60.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 302.24 | 0.00 | 657.76 |
| 60.00 | 0.48 | DSR | 0.24 | 24.00 | 960.00 | 291.02 | 0.00 | 668.98 |
| 60.00 | 0.48 | PATH | 5.24 | 24.00 | 960.00 | 271.86 | 0.00 | 688.14 |
| 60.00 | 0.48 | REG | 0.36 | 24.00 | 960.00 | 272.31 | 0.00 | 687.69 |
| 60.00 | 0.48 | DIFF | 0.30 | 24.00 | 960.00 | 288.53 | 0.00 | 671.47 |
| 120.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 128.79 | 0.00 | 831.21 |
| 120.00 | 0.48 | SLOT | 0.02 | 24.00 | 960.00 | 299.00 | 0.00 | 661.00 |
| 120.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 237.52 | 0.00 | 722.48 |
| 120.00 | 0.48 | DSR | 0.28 | 24.00 | 960.00 | 235.99 | 0.00 | 724.01 |
| 120.00 | 0.48 | PATH | 10.28 | 24.00 | 960.00 | 231.52 | 0.00 | 728.48 |
| 120.00 | 0.48 | REG | 0.54 | 24.00 | 960.00 | 223.18 | 0.00 | 736.82 |
| 120.00 | 0.48 | DIFF | 0.39 | 24.00 | 960.00 | 240.18 | 0.00 | 719.82 |
| 180.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 106.68 | 0.00 | 853.32 |
| 180.00 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 263.95 | 0.00 | 696.05 |
| 180.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 205.17 | 0.00 | 754.83 |
| 180.00 | 0.48 | DSR | 0.28 | 24.00 | 960.00 | 203.77 | 0.00 | 756.23 |
| 180.00 | 0.48 | PATH | 17.61 | 24.00 | 960.00 | 200.96 | 0.00 | 759.04 |
| 180.00 | 0.48 | REG | 0.66 | 24.00 | 960.00 | 185.57 | 0.00 | 774.43 |
| 180.00 | 0.48 | DIFF | 0.44 | 24.00 | 960.00 | 199.08 | 0.00 | 760.92 |

**Table 12** **Results for $60 \times 60$ Grid for Different Time Slot Widths**

| WIDTH | PROB | DEC TYPE | MAX TIME | STOPS | REV | DIST | FAIL | PROFIT |
|---|---|---|---|---|---|---|---|---|
| 30.00 | 0.24 | BEST | 0.00 | 18.00 | 720.00 | 393.73 | 0.00 | 326.27 |
| 30.00 | 0.24 | SLOT | 0.01 | 17.40 | 696.00 | 537.83 | 1.20 | 158.17 |
| 30.00 | 0.24 | DYN | 0.03 | 20.10 | 804.00 | 568.28 | 0.00 | 235.72 |
| 30.00 | 0.24 | DSR | 0.18 | 14.80 | 592.00 | 327.23 | 0.00 | 264.77 |
| 30.00 | 0.24 | PATH | 0.69 | 12.70 | 508.00 | 246.42 | 0.00 | 261.58 |
| 30.00 | 0.24 | REG | 0.22 | 15.20 | 608.00 | 320.26 | 0.00 | 287.74 |
| 30.00 | 0.24 | DIFF | 0.18 | 12.90 | 516.00 | 262.49 | 0.00 | 253.51 |
| 60.00 | 0.24 | BEST | 0.00 | 21.20 | 848.00 | 431.92 | 0.00 | 416.08 |
| 60.00 | 0.24 | SLOT | 0.01 | 19.80 | 792.00 | 568.86 | 0.60 | 223.14 |
| 60.00 | 0.24 | DYN | 0.02 | 23.10 | 924.00 | 588.64 | 0.00 | 335.36 |
| 60.00 | 0.24 | DSR | 0.26 | 18.10 | 724.00 | 380.77 | 0.00 | 343.23 |
| 60.00 | 0.24 | PATH | 1.51 | 16.20 | 648.00 | 323.20 | 0.00 | 324.80 |
| 60.00 | 0.24 | REG | 0.32 | 19.30 | 772.00 | 395.65 | 0.00 | 376.35 |
| 60.00 | 0.24 | DIFF | 0.24 | 14.00 | 560.00 | 269.04 | 0.00 | 290.96 |
| 120.00 | 0.24 | BEST | 0.00 | 22.70 | 908.00 | 430.69 | 0.00 | 477.31 |
| 120.00 | 0.24 | SLOT | 0.01 | 22.10 | 884.00 | 554.60 | 0.00 | 329.41 |
| 120.00 | 0.24 | DYN | 0.02 | 23.80 | 952.00 | 512.56 | 0.00 | 439.44 |
| 120.00 | 0.24 | DSR | 0.37 | 22.80 | 912.00 | 436.54 | 0.00 | 475.46 |
| 120.00 | 0.24 | PATH | 5.76 | 19.70 | 788.00 | 374.85 | 0.00 | 413.15 |
| 120.00 | 0.24 | REG | 0.56 | 23.00 | 920.00 | 432.83 | 0.00 | 487.17 |
| 120.00 | 0.24 | DIFF | 0.37 | 16.00 | 640.00 | 296.58 | 0.00 | 343.42 |
| 180.00 | 0.24 | BEST | 0.00 | 23.40 | 936.00 | 364.74 | 0.00 | 571.26 |
| 180.00 | 0.24 | SLOT | 0.01 | 21.90 | 876.00 | 507.02 | 0.00 | 368.98 |
| 180.00 | 0.24 | DYN | 0.02 | 23.90 | 956.00 | 411.99 | 0.00 | 544.01 |
| 180.00 | 0.24 | DSR | 0.47 | 23.70 | 948.00 | 403.52 | 0.00 | 544.48 |
| 180.00 | 0.24 | PATH | 14.66 | 22.70 | 908.00 | 370.40 | 0.00 | 537.60 |
| 180.00 | 0.24 | REG | 0.87 | 23.90 | 956.00 | 408.68 | 0.00 | 547.32 |
| 180.00 | 0.24 | DIFF | 0.55 | 17.70 | 708.00 | 280.06 | 0.00 | 427.94 |
| 30.00 | 0.48 | BEST | 0.00 | 23.70 | 948.00 | 392.25 | 0.00 | 555.75 |
| 30.00 | 0.48 | SLOT | 0.01 | 20.50 | 820.00 | 590.90 | 2.50 | 229.10 |
| 30.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 632.71 | 0.00 | 327.29 |
| 30.00 | 0.48 | DSR | 0.18 | 22.90 | 916.00 | 449.66 | 0.00 | 466.34 |
| 30.00 | 0.48 | PATH | 0.69 | 22.40 | 896.00 | 432.75 | 0.00 | 463.25 |
| 30.00 | 0.48 | REG | 0.22 | 23.20 | 928.00 | 475.27 | 0.00 | 452.73 |
| 30.00 | 0.48 | DIFF | 0.18 | 21.50 | 860.00 | 410.56 | 0.00 | 449.44 |
| 60.00 | 0.48 | BEST | 0.00 | 23.90 | 956.00 | 341.26 | 0.00 | 614.74 |
| 60.00 | 0.48 | SLOT | 0.02 | 22.50 | 900.00 | 627.46 | 1.50 | 272.54 |
| 60.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 634.94 | 0.00 | 325.06 |
| 60.00 | 0.48 | DSR | 0.23 | 24.00 | 960.00 | 466.32 | 0.00 | 493.68 |
| 60.00 | 0.48 | PATH | 1.66 | 23.20 | 928.00 | 410.55 | 0.00 | 517.45 |
| 60.00 | 0.48 | REG | 0.30 | 24.00 | 960.00 | 486.41 | 0.00 | 473.59 |
| 60.00 | 0.48 | DIFF | 0.25 | 23.40 | 936.00 | 443.49 | 0.00 | 492.51 |
| 120.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 265.35 | 0.00 | 694.65 |
| 120.00 | 0.48 | SLOT | 0.01 | 23.90 | 956.00 | 595.36 | 0.10 | 360.64 |
| 120.00 | 0.48 | DYN | 0.02 | 24.00 | 960.00 | 507.35 | 0.00 | 452.65 |
| 120.00 | 0.48 | DSR | 0.25 | 24.00 | 960.00 | 383.54 | 0.00 | 576.46 |
| 120.00 | 0.48 | PATH | 5.18 | 24.00 | 960.00 | 364.73 | 0.00 | 595.27 |
| 120.00 | 0.48 | REG | 0.39 | 23.90 | 956.00 | 360.94 | 0.00 | 595.06 |
| 120.00 | 0.48 | DIFF | 0.32 | 24.00 | 960.00 | 395.38 | 0.00 | 564.62 |
| 180.00 | 0.48 | BEST | 0.00 | 24.00 | 960.00 | 213.35 | 0.00 | 746.65 |
| 180.00 | 0.48 | SLOT | 0.01 | 24.00 | 960.00 | 527.90 | 0.00 | 432.10 |
| 180.00 | 0.48 | DYN | 0.01 | 24.00 | 960.00 | 457.29 | 0.00 | 502.71 |
| 180.00 | 0.48 | DSR | 0.27 | 24.00 | 960.00 | 340.19 | 0.00 | 619.81 |
| 180.00 | 0.48 | PATH | 9.45 | 24.00 | 960.00 | 322.52 | 0.00 | 637.48 |
| 180.00 | 0.48 | REG | 0.56 | 24.00 | 960.00 | 352.10 | 0.00 | 607.90 |
| 180.00 | 0.48 | DIFF | 0.40 | 24.00 | 960.00 | 337.78 | 0.00 | 622.22 |

(going from 3.26 to 9.18 to 29.76 to 50.70 seconds). For the $60 \times 60$ grid, there are substantial improvements for the SLOT method, but the profit values are still not close to those obtained with the feasibility- and profitability-based insertion criteria due to the larger customer to customer distances. Note that there are still some missed guaranteed deliveries with 2-hour (120 minute) windows for the SLOT method, which indicates that feasibility-based insertion methods already provide a significant improvement over current practice when customers are fairly spread out. It does not appear that the window width affects the performance of the profitability-based methods compared to the best possible (BEST), because the ratio of PATH profit to BEST profit, when expected demand to capacity ratio is 2, is 0.83, 0.84, 0.85, and 0.85 respectively, which is amazing consistency.

### 6.3. Insights

Our preliminary computational experience suggests that the profitability of HDSs can be improved significantly by the use of more sophisticated approaches to order capture and promise. More specifically, our computational experience indicates the following.

- Dynamically evaluating the feasibility of a delivery in a given time slot (as opposed to limiting the number of deliveries in a time slot to a fixed number) can significantly enhance profitability and clearly reduce the risk of failures.
- The value of more sophisticated approaches increases as the expected demand to capacity ratio increases.
- The value of more sophisticated approaches increases as customer density decreases.
- The width of the time slots has a significant impact on the profitability even for dense areas.
- The number of slots in an average customer's time slot profile has an impact on profitability.
- It is advantageous to have more, smaller vehicles rather than fewer, larger vehicles, as that increases the ability to satisfy desired delivery windows. In practice, where there are several high-demand time periods, e.g., early evening, this is extremely important.
- The ability of our proposed profitability-based methods to capitalize on revenue differences among customers depends both on probabilities (variability in expected revenues) and distances between customers (variability in delivery costs).
- Our proposed profitability-based methods PATH and REG perform better in denser areas.
- Our proposed profitability-based method PATH captures the tradeoff between accepting (and realizing) a low-profit request and waiting for (and possibly missing) a high-profit request well but requires a substantial computational effort in certain situations. Improved implementations may alleviate this

somewhat, but the method may prove to be impractical. Methods DSR and DIFF are not far behind in terms of quality and display more favorable computational behavior.

## 7. Future Directions and Conclusions

Our computational experiments demonstrate that with a little intelligence the cost effectiveness (and thus profitability) of HDS can be significantly enhanced. This in itself is an important result. However, in our opinion, there is an opportunity for even greater improvements by incorporating profit- and revenue-optimization concepts and techniques in the HDS environment.

Most HDSs offer the customer the ability to select a time slot for the delivery. It is intuitively clear that the closer the delivery locations are for orders in a given time slot (or consecutive time slots), the easier it will be to schedule the deliveries and the cheaper it will be to carry them out. However, without incentives there is little motivation for consumers to choose time slots that are convenient for the retailer. Peapod (www.peapod.com) already offers $1 and $2 discounts for allowing larger delivery windows and has found this to be successful (Fendelman 2001). We propose to use dynamic incentive schemes. Altering the consumer choices—e.g., by restricting the number of delivery options or by *pricing* the time slots differently—may help the retailer build better, more cost-effective schedules.

We expect that the next generation of decision support systems for HDS will integrate dynamic routing and scheduling and profit- and revenue-optimization techniques into time-slot management systems that will be responsible for dynamically presenting time slots to customers based on the currently available, tentative delivery schedule and the customer's order and location. To even out the distribution of demand, time slots may be accompanied by price incentives (discounts) to encourage customers to choose particular delivery windows (which are better from a scheduling perspective) or to accept wider delivery windows (which give more flexibility in scheduling the delivery). These incentives will be based on the opportunity cost calculations that reflect how well a delivery at the customer's location in a specific time slot fits into the tentative schedule.

To be able to study the integration of dynamic routing and scheduling and profit- and revenue-optimization techniques for HDS, we propose the home delivery problem with incentives (HDPI). Instead of a time slot profile for each customer identifying which time slots are acceptable for delivery, there is a *time slot revenue profile* for each customer. This profile specifies the price a customer is willing to pay for a delivery in a particular time slot. This price can be thought of as the price of purchased goods plus the price the customer is willing to pay for delivery at a particular time (note that by allowing the "delivery price" to be negative it is possible to model undesirable or even infeasible time slots). When a delivery request arrives, it must either be rejected or accepted, and, if accepted, a commitment has to be made to a specific time slot for delivery. The objective is to maximize the total profit resulting from executing the set of delivery routes, i.e., total revenues minus total costs.

This problem can also be viewed as follows. The basic price a customer pays for a home delivery is the price of the purchased goods and a fixed delivery charge. When a delivery request arrives, the HDS provider displays the set of viable time slots along with delivery discounts associated with certain slots. The customer selects one or decides to withdraw the delivery request.

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### References

Bagchi, P. K., B. N. Nag. 1991. Dynamic vehicle scheduling: An expert systems approach. *Internat. J. Physical Distribution Logistics Management* **21**(2) 10–18.

Bertsimas, D. 1992. A vehicle-routing problem with stochastic demand. *Oper. Res.* **40**(3) 574–585.

Bertsimas, D., D. Simchi-Levi. 1996. A new generation of vehicle routing research: Robust algorithms, addressing uncertainty. *Oper. Res.* **44**(2) 286–303.

Bertsimas, D., P. Jaillet, A. Odoni. 1990. A priori optimization. *Oper. Res.* **38**(6) 1019–1033.

Cheung, R. K., W. Powell. 1996. An algorithm for multistage dynamic networks with random arc capacities with an application to dynamic fleet management. *Oper. Res.* **44**(6) 951–963.

Cox, B. 2001. Internet grocer seeks an angel. *InternetNews* (April 16) www.internetnews.com.

Farmer, M., G. Sandoval. 2001. Webvan delivers its last word: Bankruptcy. *News.com* (July 9) www.news.com.

Fendelman, A. 2001. First time, first market: Peapod achieves profitability in Chicago. *Eprairie.com* (April 24) www.eprairie.com.

Frantzekakis, L. F., W. B. Powell. 1990. A successive linear approximation procedure for stochastic, dynamic vehicle allocation problems. *Transportaion Sci.* **24** 40–57.

Gendreau, M., G. Laporte, R. Seguin. 1995. An exact algorithm for the vehicle routing problem with stochastic demands and customers. *Transportation Sci.* **29**(2) 143–155.

Gendreau, M., G. Laporte, R. Seguin. 1996. Stochastic vehicle routing. *Eur. J. Oper. Res.* **88** 3–12.

Jardine Logistics. 2001. The challenge of e-logistics. Report, Jardine Logistics.

Kontoravdis, G., J. Bard. 1995. A grasp for the vehicle routing problem with time windows. *ORSA J. Comput.* **7** 10–23.

Littlewood, K. 1972. Forecasting and control of passengers. *12th AGIFORS Sympos. Proc.*, New York, 95–117.

Powell, W. 1996. A stochastic formulation of the dynamic assignment problem with an application to truckload motor carriers. *Transportation Sci.* **30** 195–219.

Prosser, P., P. Kilby, P. Shaw. 1998. Dynamic VRPs: A study of scenarios. Technical Report APES-06-1998, University of Strathclyde.

Psaraftis, H. 1988. Dynamic vehicle routing problems. B. L. Golden, A. A. Assad, eds. *Vehicle Routing: Methods and Studies*. Elsevier Science Publishers, Amsterdam, The Netherlands, 223–249.

Psaraftis, H. 1995. Dynamic vehicle routing: Status and prospects. *Ann. Oper. Res.* **61** 143–164.

Punakivi, M. 2000. The value added of route optimisation in home delivery. Working paper, The ECOMLOG Research Program, Department of Industrial Engineering and Management, Helsinki University of Technology, Helsinki, Finland.

Regan, A., H. Mahmassani, P. Jaillet. 1998a. Dynamic decision making for commercial fleet operations using real-time operations. *Transportation Res. Record* **1537** 91–97.

Regan, A., H. Mahmassani, P. Jaillet. 1998b. Evaulation of dynamic fleet management systems. *Transportation Res. Record* **1645** 176–184.

Rego, C., C. Roucairol. 1995. Using tabu search for solving a dynamic multi-terminal truck dispatching problem. *Eur. J. Oper. Res.* **83** 411–429.

Saranen, J., J. Småros. 2001. An analytical model for home delivery in the new economy. Working paper, The ECOMLOG Research Program, Department of Industrial Engineering and Management, Helsinki University of Technology, Helsinki, Finland.

Savelsbergh, M. 1986. Local search for routing problems with time windows. *Ann. Oper. Res.* **4** 285–305.

Savelsbergh, M., M. Sol. 1998. Drive: Dynamic routing of independent vehicles. *Oper. Res.* **46** 474–490.

Talluri, K., G. van Ryzin. 1998. An analysis of bid-price controls for network revenue management. *Management Sci.* **44** 1577–1593.

Talluri, K., G. van Ryzin. 1999. A randomized linear programming method for computing network bid prices. *Transportation Sci.* **33**(2) 207–216.

Yrjölä, H. 2001. Physical distribution considerations for electronic grocery shopping. *Internat. J. Physical Distribution Logistics Management* **31**(10) 746–761.

Zhu, K., K.-L. Ong. 2000. A reactive method for real time dynamic vehicle routing problems. *12th IEEE Internat. Conf. Tools Artificial Intelligence*, Vancouver, BC, Canada (November).